FINAL REPORT

ON

THE STUDY OF THE RELATIONSHIP

BETWEEN

PROBABILISTIC DESIGN
AND
AXIOMATIC DESIGN METHODOLOGY

Submitted By

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Appendix D

Copies of Masters Theses.

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1. System Reliability Studies of a Plane Frame Single Story Structure Under Cumulative

   Damage.

2. Study of design of a gear train using reliability method based on optimization design method.

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SYSTEM RELIABILITY STUDIES OF A PLANE FRAME SINGLE

STORY STRUCTURE UNDER CUMULATIVE DAMAGE

A Project

Submitted to the College

of

Engineering and Technology

in

Partial Fulfillment of the Requirements

for the Degree of

Master of Engineering

with a

Civil Engineering Option

Nitish Beri

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CHAPTER I

INTRODUCTION

Structural failure is rarely a "sudden death" type of event, such sudden failures may occur only under abnormal loadings like bomb or gas explosions and very strong earthquakes. In most cases, structures fail due to damage accumulated under normal loadings such as wind loads, dead and live loads. The consequence of cumulative damage will affect the reliability of surviving components and finally causes collapse of the system. The cumulative damage effects on system reliability under time-invariant loadings are of practical interest in structural design and therefore will be investigated in this study.

The scope of this study is, however, restricted to the consideration of damage accumulation as the increase in the number of failed components due to the violation of their strength limits. Progressive failure processes such as corrosion, fatigue and crack growth are not investigated in this study.

1.1 Background and Significance of the Study

Structural designs have been traditionally based on *deterministic design* methodology. The deterministic method considers all design parameters to be known with certainty. This methodology is, therefore, inadequate to design complex structures subjected to a variety of complex, severe loading conditions. These complex conditions introduce uncertainties and so the actual factor of safety remains unknown. In the deterministic methodology the contingency of failure is discounted, so there is a use of a high factor of safety.

Probabilistic design method is concerned with the probability of non-failure performance of structures or machine elements. Probabilistic methodology is a convenient tool to describe, or model, physical phenomena too complex to treat with the present level of scientific knowledge. It is much more useful in situations where the design is characterized by complex geometry, possibility of catastrophic failure or sensitive loads and material properties.

1.1.1 Comparison between Deterministic and Probabilistic Design Methodology

The probabilistic design methodology produces designs that are robust and allows the quantification of the level of reliability in the design, as opposed to

deterministic designs. Hence, it is begining to attract more attention than the traditional deterministic design.

Probabilistic design procedures promise to improve the quality of engineered systems for the following reasons:

1. Probabilistic design incorporates given statistical data explicitly into design algorithms. Conventional design discards such data.

2. It is more meaningful to say, "This system has a probability of 10^{-4} of failing after 1000 hours of operation," than to say, "This system has a factor of safety of 2.3."

3. Rational comparisons can be made between two or more competing designs for a proposed system. Without other considerations, the engineer chooses the design having the lowest probability of failure, or basis for developing economic strategies.

4. An "optimal" design of a system results when each component chosen so that its probability of failure is the same.

5. By treating each nonstatistical uncertainty as a random variable, its effect on the final design can be quantified.

6. Probabilistic-based information on mechanical and structural performance can be used to develop rational

policies toward pricing, warranties, etc.

1.1.2 Structural Reliability under Time-invariant Loads

This study primarily focusses on the effects of time-invariant loads on the structure. The effects of time-invariant loads on element and system reliability are discussed below.

1.1.2.1 Element Reliability

The study of element reliability under cumulative damage is to include the system effects into element reliability. In the current codes such as CEB[1], LRFD[2] and AASHTO[3] specifications, the design of a structural system goes through the design of components and connections individually. The target element reliability and safety are achieved by making them satisfy the limit state functions of local strength with a high degree of probability. What is the reliability of the individual component once it is in the actual configuration? How do the system effects influence the element reliability and which components are more vulnerable than the others? What impact do these questions have on a current reliability-based design code, like the AISC load and resistance factor design (LRFD) code? These are some preliminary questions sought to be answered in this study.

Mahadevan and Haldar[4] used the Stochastic Finite

Element Method (SFEM) to investigate the magnitude of system effects on component reliability in framed structures designed by the LRFD code. However, their analysis was based on linear elastic behavior. The effect of geometric non-linearity was included in SFEM-based reliability analysis by Liu and Der Liureghian[5] and Haldar and Zhou[6]. The effect of material nonlinearity has been considered by many researchers to estimate the overall system reliability, but the focus of this study is the reliability of individual elements affected by the formation of plastic hinges elsewhere in the structure. Therefore, a rational procedure has been developed in this study to account for the effect of structural system damage.

Although system reliability research has been active for the past twenty years, it still has not been applied in practical design. The inclusion of system effects on element reliability may offer a solution to this problem so that the element-based design can account for system effects.

1.1.2.2 System Reliability

The collapse of a system is the culmination of cumulative damage of components. This idea resulted in the development of several failure path identification techniques including branch and bound method, β-unzipping method, etc. However, these techniques are difficult to implement in case of large

structures, because they are time consuming. This is one of the important reasons for the slow application of system reliability in modern design[7]. This study and analysis are used to examine the performance of the LRFD approach in the design of realistic structures, resulting in several important observations.

In this study, the loadings are idealized as time-invariant. In other words, the reliability so obtained corresponds to that under one load application though it may represent some extreme value of the load over a given period. However, the reliability of an element or a structure varies over its lifetime, due to repetitive load applications causing accumulated damage, degradation of material resistance over time, corrosion, wear etc.

1.2 Research Objectives and Organization of the Report

The above discussion leads to the following objectives:

1. Discussion of the probabilistic design methodology in depth and an overview of the software, NESSUS(Numerical Evaluation of Stochastic Structures Under Stress) used primarily in this project. This is described in detail in Chapter II.

2. Discussion of the LRFD source codes and their

application to this project. This is described in Chapter III.

3. Development of a failure path-based procedure to estimate system reliability of assumed steel structures, along with the development of a computational procedure to estimate the element reliability under cumulative damage. This is described in Chapter IV.

4. The results of the system reliability analysis, their interpretation and explanation, are described in Chapter V.

5. The summary and conclusions of the present study and suggestions for further research are presented in Chapter VI.

6. The appendix A lists the computer program formulated for lognormal distribution. Appendix B gives the detailed loading calculations done for the structures in accordance with the Uniform Building Codes. Moment analysis of the structures, which is done by finite element software(STAAD-III) is given in appendix C. The algorithm and flowchart to operate NESSUS for probabilistic design is listed in Appendix D.

CHAPTER II

PROBABILISTIC DESIGN METHODOLOGY

2.1 Role of Probability in Engineering

Quantitative methods of modeling, analysis, and evaluation are the tools of modern engineering. Some of these methods have become quite elaborate and include sophisticated mathematical modeling and analysis, computer simulation, and optimization techniques. However, irrespective of the sophistication in the models, including experimental laboratory models, they are predicated on idealized assumptions or conditions; therefore, information derived from these quantitative models may or may not reflect reality closely.

In engineering designs, decisions are often required irrespective of the state of completeness and quality of information, and thus are made under conditions of uncertainty. In other words the consequence of a given decision cannot be determined with complete confidence. Besides the fact that the information must often be inferred from similar circumstances or derived through modeling. Many problems in engineering involve natural processes and phenomena that are inherently random; the states of such

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phenomena are naturally indeterminate and **thus** cannot be described with definiteness. For these **reasons,** decisions required while engineering planning and design **invariably** must be made, and are made, under conditions of uncertainty.

The effects of such uncertainty in design **and** planning are important. To be sure, the quantification of such uncertainty and evaluation of its effects on **the** performance and design of an engineering system, should include concepts and methods of probability. Further, under conditions of uncertainty, the design and planning of engineering systems involve risks, and the formulation of related decisions requires them to be risk free. The problems of uncertainty in the design can be overcome by applying the methods of probability. Thus, the role of probability is quite pervasive in engineering. It ranges from the description of information to the development of bases for design and decision making[8]. Many phenomena or processes of concern to engineers contain randomness, that is, the actual outcomes are sometimes unpredictable. Such phenomena are characterized by experimental observations that are different from one experiment to another, even if performed under identical conditions. In other words, there is usually a range of measured or observed values and within this range certain

values may occur more frequently than others. Clearly, if recorded data is of a variable exhibit scatter **or dispersion**, the value of the variable cannot be predicted **with** certainty. Such a variable is known as **random variable** and its value or range of values can be predicted only with an associated probability. When two or more random variables **are** involved, the characteristics of one variable may depend on the other.

Since there is a range of possible values of random variable, we would be interested in some central value, such as the average. In particular, because the different values of the random variable are associated with different probabilities, the weighted average is taken into consideration. This weighted average is known as **sample mean** value of the random variable. Therefore, if X is a discrete random variable, then the mean value, μ_x is obtained as follows

$$\mu_x = \frac{\Sigma X}{n} \qquad (2\text{-}1)$$

where,

μ_x is the mean

X is the random variable.

n is the number of observations.

Besides the sample mean, the next most important quantity of a random variable is its measure of **dispersion** or variability, that is, the quantity that gives a measure of how widely the values of the variate are spread around its mean value. This deviation can be above or below its central value. If the deviations are taken with respect to its mean value, then a suitable average measure of dispersion is called the

$$Var(X) = \frac{\Sigma(X - \mu_x)^2}{n - 1} \qquad (2\text{-}2)$$

variance and is computed using the following relation:
where,

Var(X) is the variance of the random variable X.

Dimensionally, a more convenient measure of dispersion is the square root of the variance, or the **standard deviation,**

$$\sigma_x = \sqrt{Var(X)} \qquad (2\text{-}3)$$

where,

σ_x is the standard deviation of the random variable X.

Saying whether the dispersion is large or small is difficult, from the variance or standard deviation. For this purpose, the measure of dispersion about the central value is

more useful. In other words, the dispersion is large or small is meaningful only about the central value. Therefore, **coefficient of variation** (COV) is often preferred, which is a convenient non-dimensional measure of dispersion or variability. The coefficient of variation is related to the mean and standard deviation as follows,

$$COV = \frac{\sigma_x}{\mu_x}$$ (2-4)

where,

σ_x = Standard deviation of the variable X.

μ_x = Mean value of the variable X.

The application of probability is not limited to the description of experimental data, or the evaluation of the statistics such as the mean and standard deviation. In fact, the more significant role of probability concepts is in the use of this information in the formulation of proper bases for the design.

2.2 Uncertainty Associated with Design

Engineering uncertainty is not limited to the variability observed in the basic variables. First, the estimated values of a given variable (such as the mean) based on observational

data will not be error free. Second, the mathematical or simulation models (for example, formulas, equations, algorithms and laboratory models, that are often used in engineering analysis and designs are idealized representations of reality). Consequently, predictions and calculations made from these models may be inaccurate (to some unknown degree) and thus also contain uncertainty. Human error can result from errors made by engineers and technicians during the design or operations phases. It can be reduced by improving the quality of a control program, but it cannot be avoided entirely. Usually, human error is very difficult to define. In study, human error will be treated as modeling error. In some cases, the uncertainties associated with such prediction or model errors may be much more significant than those associated with the inherent variabilities.

All uncertainties, whether they are associated with inherent variability or with prediction error, may be assessed in statistical terms and the evaluation of their significance on the design can be accomplished by the concepts and the methods of probability.

2.3 Designing under Uncertainty

If there are uncertainties in the design, the next step is to ask, how should designs be formulated or decisions affecting a design resolved? Presumably we may assume the worst conditions and develop conservative design on this basis. From the system performance and safety point of view, this approach may be suitable. However, the resulting design would be too costly because of over conservatism. On the other hand an inexpensive design may not ensure the desired level of performance and safety. Therefore the decisions should be made considering cost and safety of the design. The most desirable solution is one that is optimal, in the sense of minimum cost and maximum benefits. If the available information and the models to be evaluated contain uncertainties, the analysis should include the effects of such uncertainties.

Probabilistic design is concerned with the probability of failure, or preferably, reliability. This methodology is most useful when uncertainties in material properties and loading conditions are considered. To apply probabilistic design methodologies(PDM), all uncertainties are modeled as random variables, with selected distribution types, means and standard deviations. The primitive (random) variables that affect the structural behavior have to be identified. Every

design project demands some sequential stages of reflection before one can arrive at the final design goal. This is also the case with PDM. The various design stages of PDM are as follows.

1. Problem Definition.

2. Generating design parameters.

3. Relating the defined problem to the design parameters.

4. Data assembling and application of probability concepts.

5. Probabilistic Analysis.

6. Interpreting results.

The design stages of PDM are shown in Figure 2-1.
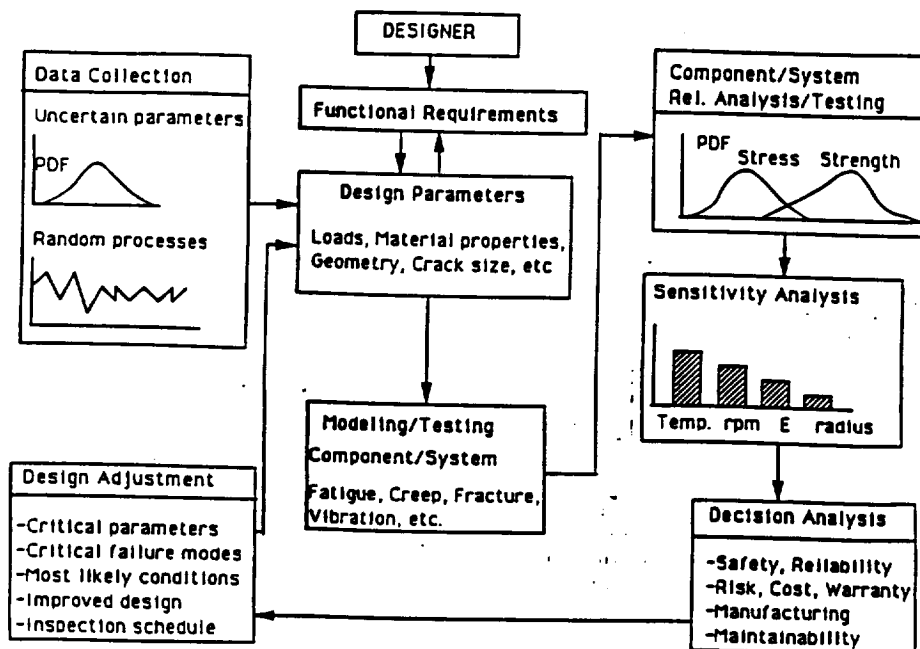
Figure 2-1: Design stages in PDM [9]

1. Problem definition

The first step a designer takes in solving a design problem is to find out the main objective of the design. After finding out the objective, the next step is to define in a precise manner the functional requirements, of the system or component to be designed. These functional requirements should be able to completely characterize the design objective by defining it in terms of specific needs. With a clear understanding of what one is searching for, the designer then goes to the next stage.

2. Generating design parameters

In order to solve the defined problem, acceptable design parameters must be generated that will meet the defined functional requirements. To generate the design parameters one uses an appropriate design model. The various parameters like loads, material properties, geometry, crack size etc. are taken into consideration. The design parameters to be selected depend on the objective of the design[9].

3. Relating the defined problem to the design parameters

After defining the design parameters the designer then relates the functional requirements in the functional domain

to the design parameters in the physical domain, to be sure that the objective is satisfied. If the relation is satisfactory, the designer goes to the next stage, if not the relation is redefined, so that the objective is satisfied.

4. Data assembling and application of probability concepts

This stage requires assembling the essential data that are available on the problem with regard to the design parameters. If some data are unavailable then it becomes necessary to perform a computational simulation analysis to generate the missing details. Once the data has been assembled, the next stage is to analyze the assembled data. NESSUS is the computer tool used to perform the analysis. NESSUS has three modules known as NESSUS/PRE, NESSUS/FEM and NESSUS/FPI.

NESSUS/PRE is a preprocessor, which prepares the statistical data needed for the probabilistic design analysis. It allows the user to describe the uncertainties in the structural design parameters. The uncertainties in these parameters are specified by defining the mean value, standard deviation and the distribution type, together with an appropriate form of correlation. Correlated random variables are then decomposed into a set of uncorrelated vectors by a

model analysis.

NESSUS/FEM is a general purpose finite element code, which is used to perform structural analysis and evaluation of sensitivity due to variation in different uncorrelated random variables. The response surface, defined in terms of random variables required for probabilistic analysis in NESSUS/FPI, is obtained from NESSUS/PRE. NESSUS/FEM incorporates an efficient perturbation algorithm to compute the sensitivity of random variables [10].
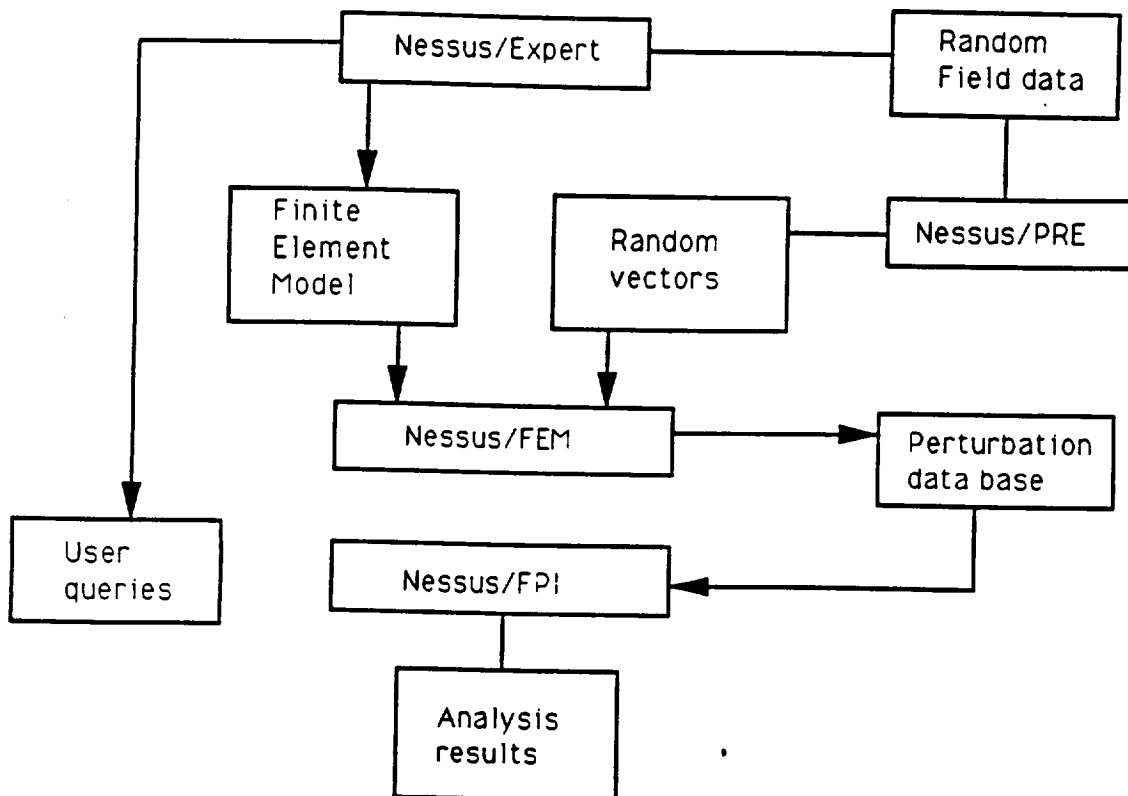
Figure 2-2: Modules of NESSUS

NESSUS/FPI is an advanced reliability module, which

extracts the database generated by NESSUS/FEM to develop a response model in terms of random variables[11]. In this module the probabilistic structural response is calculated from the performance model. The probability of exceeding a given response value is estimated by a reliability method. Inside the NESSUS/FPI module is a sensitivity analysis program, which determines the most critical design parameters in the design. The input data for NESSUS/PRE requires fundamental knowledge of statistics or probability theorems. The expected details will include determining the mean, standard deviation, median, coefficient of variation, variances etc., associated with each random variable. The designer also determines the probability distribution function that best describes each random variable. The different modules of NESSUS are shown in Figure 2-2.

5. Probabilistic Analysis

It is at this stage of the design that the designer defines a limit state function. The limit state function is a function that defines the boundary between the safe and failure region. In the limit state function approach for structural reliability analysis, a limit state function $g(X)$ is first defined. The g-function, is a function of a vector of

basic random variables, $\mathbf{X}=(X_1, X_2, X_3, ...X_n)$ with $g(\mathbf{X}) = 0$
being the limit state surface that separates
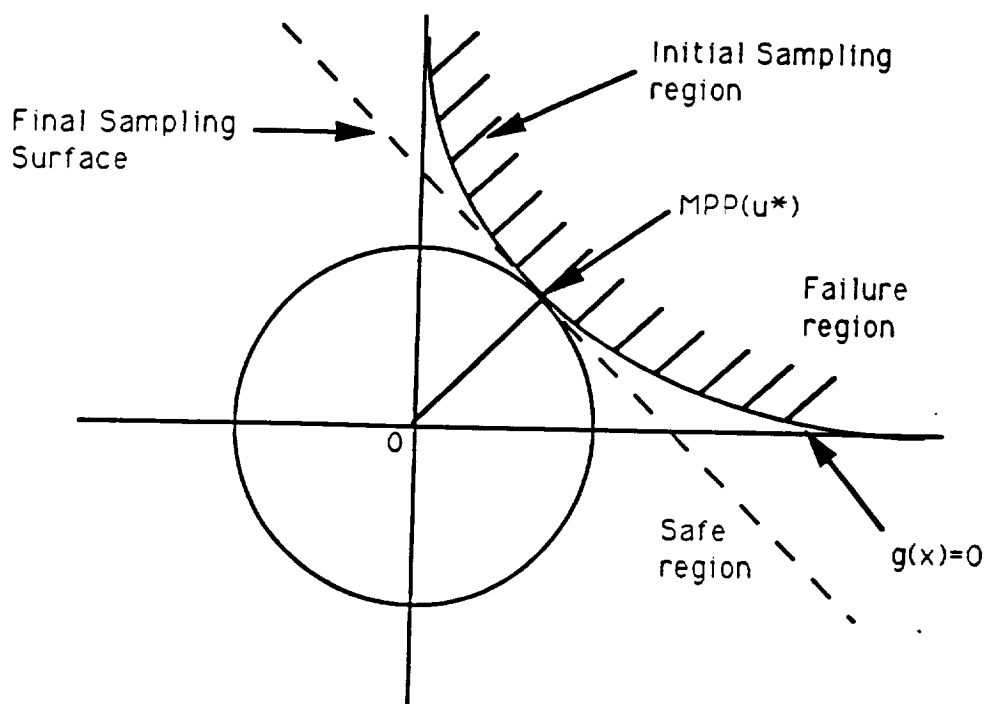
Figure 2-3: Illustration of Most Probable Point

the design space into two regions, namely, the failure
$g(\leq 0)$ and the safe $g(>0)$ regions. Geometrically, the limit

state equation, g(X)=0, is a n-dimensional surface that may be called the "failure surface." One side of the failure surface is the safe state, g(X)>0, whereas the other side of the failure surface is the failure state, g(X)<0.

The probability of failure in the failure domain Ω is given by;

$$P_f = \int_\Omega \ldots \int f_x(X) \ dx \qquad (2-6)$$

where $f_x(X)$ is the joint probability density function of X and Ω is the failure region. The solution of this multiple integral is, in general, extremely complicated. Alternatively, a Monte Carlo solution provides a convenient but usually time consuming approximation. The limit state function method uses the Most Probable Point (MPP) search approach shown in Figure 2-3. The Most Probable Point is the key approximation point for the FPI analysis, therefore, the identification of MPP is an important task. In general, the identification of the MPP can be formulated as a standard optimization problem and solved by proper optimization methods.

From the Figure 2-3, as the limit state surface g(X)=0, moves closer to the origin, the safe region, g(X)>0, decreases accordingly. Therefore, the position of the failure surface relative to the origin of the reduced variates should determine the safety or reliability of the system. The

position of the failure surface may be represented by the minimum distance from the surface g(X)=0 to the origin. The point on the surface with minimum distance to the origin is the Most Probable Point (MPP). This is usually determined by fitting a local tangent to g(X) and moving this tangent until MPP is estimated.

In the NESSUS code MPP is defined in a transformed space called u-space where the u's are independent to simplify the probability computations. By transforming g(x) to g(u), the most probable point, u^*, on the limit state, g(**X**)=0, is the point that defines the minimum distance from the origin to the limit state surface. This point is most probable (in the u-space) because it has maximum joint probability density on the limit state surface. The required minimum distance is determined as follows. The distance from a point $u^*=(u_1^*, u_2^*, ..., u_n^*)$ on the failure surface g(u)=0 to the origin is,

$$D = \sqrt{u_1^{*2} + u_2^{*2} + ... + u_n^{*2}} \qquad (2\text{-}7)$$

where, D is the minimum distance from the point on the limit state surface to the origin.

The FPI code assumes only one MPP. In general, however,

the possibility exists that there may exist multiple local and global Most Probable Points. A two MPP problem can occur for example, if the g-function is quadratic and the search algorithm may result in an oscillating (non-convergent) search.

Several approaches are available to search for the MPP. The search procedure depends on the forms and the number of the g-function(s). One efficient method in use is the Advanced Mean Value method. This method blends the conventional mean value method with the advanced structural reliability analysis method. This method provides efficient cumulative density function analysis and the reliability analysis. The step wise AMV method can be summarized as follows [12]:

1. Obtain the g(X) function based on perturbations about the mean values.

2. Compute the cumulative density function of the performance function at selected points using the fast probability integration method.

3. Select a number of cumulative density function values that cover a sufficiently wide probability range.

4. For each cumulative density function value, identify the most probable point.

Another approach considered efficient as well is the

Adaptive Importance Sampling Method. This method focusses on reducing the sampling domain in the search space after the MPP is identified. The **Adaptive Importance Sampling** method is generally used for **system reliability analysis.**

The analytical process involved in the limit state approach can be illustrated by a basic structural reliability problem. In the problem only one load effect S, restricted by one resistance R, is considered.

If one considers a case when R and S are independent, the limit state equation can be expressed as,

$$g = R - S \qquad\qquad (2-8)$$

and the probability of failure can be expressed as,

$$P_f = P(R-S \le 0) = \int_{-\infty}^{\infty} \int f_R(r) \, f_S(s) \, dr \, ds \qquad (2-9)$$

For any random variable the cumulative density function F(x), is given by

$$F_x(x) = P(X \le x) = \int f_x(y) \, dy \qquad (2-10)$$

provided that x ≥ y

Therefore P_f is expressed as

$$P_f = P(R-S \le 0) = \int F_R(x) \, f_S(x) \, dx \qquad (2-11)$$

Assuming a special case of normal random variables, for some distributions of R and S, it is possible to integrate the equation (2-11) analytically and find out the probability of

failure. If S and R have mean μ_R and μ_S and variance's σ_R^2 and σ_S^2 respectively, the g-function has a mean μ_g and variance σ_g^2, given by

$$\mu_g = \mu_R - \mu_S \qquad (2\text{-}12)$$

$$\sigma_g^2 = \sigma_R^2 + \sigma_S^2 \qquad (2\text{-}13)$$

Therefore the probability of failure is given as,

$$P_f = P(R - S \leq 0) = P(g \leq 0) = \phi[\frac{0 - \mu_g}{\sigma_g}] \qquad (2\text{-}14)$$

$$\Phi[- \frac{(\mu_R - \mu_S)}{\sqrt{(\sigma_S^2 + \sigma_R^2)}}] = \Phi(-\beta) \qquad (2\text{-}15)$$

Which reduces to,

$$\beta = \frac{\mu_g}{\sigma_g} \qquad (2\text{-}16)$$

where β is defined as the safety index.

Thus the probability of failure is given as

$$P_f = \phi(-\beta) \qquad (2\text{-}17)$$

which can be written as,

$$P_f = 1 - \phi(\beta) \qquad (2\text{-}18)$$

The reliability of the system is given by

$$P_r = 1 - P_f \qquad (2\text{-}19)$$

where P_r is the reliability of the system.

6. Interpretation of Results

This is the last stage in the methodology. When the designer approaches this stage, one interprets the results obtained about the initial objective. If the results do not satisfy the functional requirements in the stage 1, the designer may adjust order to achieve the set objective.

2.4 Probability Sensitivity Factors

In Engineering performance analysis many sensitivity measures can be defined. Knowing the effect of each random variable in the analysis is important for the designer. The

sensitivity information is quantified by sensitivity factors. Sensitivity factors suggest which random variables are crucial and require special attention.

The commonly used sensitivity in deterministic analysis is the performance sensitivity, $\partial Z/\partial X_i$, which measures the change in the performance due to the change in a design parameter. This concept can be extended to the probabilistic analysis in which a more direct sensitivity measure is the reliability sensitivity that measures the change in the probability/reliability relative to the distribution parameters such as the mean and the standard deviation. Although not automated in the code, this analysis can be performed by varying the parameters.

Another, perhaps more important, kind of probability or reliability sensitivity analysis is the determination of the relative importance of the random variables. This analysis can be done, for example, by repeated probabilistic analysis in which one random variable at a time is treated as a deterministic variable. The results of the analyses, for example, are a number of cumulative density function curves or reliabilities. Based on the results, the relative importance of the random variables can be analyzed. The standard FPI output includes a first order sensitivity factor that provides

approximate relative importance of the random variables.

CHAPTER III

LOAD AND RESISTANCE FACTOR DESIGN FOR STEEL

Inherent uncertainties in structural design parameters, such as loads, geometry, material and sectional properties, and boundary conditions, are well established. However, in traditional design procedures, these parameters are considered deterministic; the uncertainty is accounted for by the use of safety factors. Thus, in allowable-stress design method, the ultimate stresses are divided by safety factors to determine the allowable stresses. A successful design ensures that the stresses caused by the nominal values of the loads do not exceed the allowable stresses. In the ultimate-strength, or plastic, design, the loads are multiplied by the load factors to determine the ultimate loads and the fully stressed members are required to resist various design combinations of these ultimate loads[13].

A more rational approach to consideration of stochasticity in structural parameters has resulted in the development of the LRFD approach during the past decade.

3.1 General Discussion on LRFD codes

The load and resistance factor design criterion is

expressed by the following general formula:

$$\Phi R_n \geq \sum \gamma_i Q_i \qquad\qquad (3\text{-}1)$$

The left side of the formula relates to the resistance (capacity) of the structure while the right side characterizes the loading acting on it.

The resistance side of the design criterion consists of the product ΦR_n, in which R_n is the "nominal resistance," and Φ is the "resistance factor." The nominal resistance is the resistance computed according to a formula in a structural code and it is based on the nominal material and cross-sectional properties. The resistance factor Φ, which is always less than unity, together with R_n reflects the uncertainties associated with R. The factor Φ is dimension less and R_n is a generalized force: bending moment, axial force, or shear force associated with a limit state of strength and serviceability. Interaction equations, e.g., between axial force and bending, may also be used to define R_n for appropriate limit states. The loading side of the design criterion is the sum of products, $\gamma_i Q_i$, in which Q_i is the "mean load effect," and γ_i is the corresponding "load factor." Here γ_i is dimensionless and Q_i is a generalized force (i.e., bending moment, axial force or shear force) computed for the mean loads for which the structure is to be designed. The γ-factors reflect

potential overloads and uncertainties inherent in the calculation of the load effects. The summation sign in the equation denotes the combination of load effects from different load sources[13].

The LRFD codes were developed, based on first order probabilistic design methods. In LRFD, the nominal resistance always relates to a specific "limit state." Two classes of limit states are pertinent to structural design: the "maximum strength"(or "ultimate")limit state, and the "serviceability" limit state. Violation of a strength limit state implies "failure" in the sense that a clearly defined limit of structural usefulness has been exceeded, but this does not necessarily involve actual collapse. In case of structural system with "compact" beams this means that a plastic mechanism has formed. Serviceability limit states include excessive deflection, excessive vibration, and premature yielding or slip.

A first order probabilistic design procedure was used to determine the values of Φ, R_n, γ and $_iQ$, during the development of the code. This is simplified method that uses only statistical parameters, i.e., means values and coefficients of variation of relevant variables and a relationship β between them, called the "safety index."

Probability-based LRFD criteria have been adopted in Canada for hot-rolled and cold-formed steel structures, the basic guidelines for European national codes have been formulated, and research on development of similar procedures is underway for reinforced concrete and wood structures. Experience gained from one effort is transmitted to newer projects, and the concepts of the applications of probability, statistics, optimization, and decision theories have become increasingly more sophisticated[13]. Thus the field of design methodology research is very active and changes occur rapidly.

3.2 Selection of Model

The probabilistic design format used to develop the LRFD criteria for steel structures is due to Cornell[14]. This format was selected because of its simplicity and its ability to treat all uncertainties in a design problem in a consistent manner. The format is explained briefly in the following.

Structural safety is a function of resistance, R, of the structure and of the load effect, Q, acting on it; R and Q are random variables. An example of the definition of safety is given in the Figure 3-1, where the frequency distribution of the random variable of R-Q, called the safety margin, is shown and survival is indicated by R-Q, called the safety margin, is

shown and survival is indicated by R-Q > 0. The probability of failure p_F of a structural element according to the representation of Figure 3-1 is equal to

$$\overline{p_F} = P[(R-Q) < 0] \qquad (3-2)$$

An equivalent representation of structural safety is shown in figure where the probability of failure is

$$p_F = P[\ln(R/Q) < 0] \qquad (3-3)$$

The format according to the Figure 3-1 was adopted for developing the LRFD criteria.

(a) PROBABILISTIC MODEL   (b) DEFINITION OF SAFETY INDEX
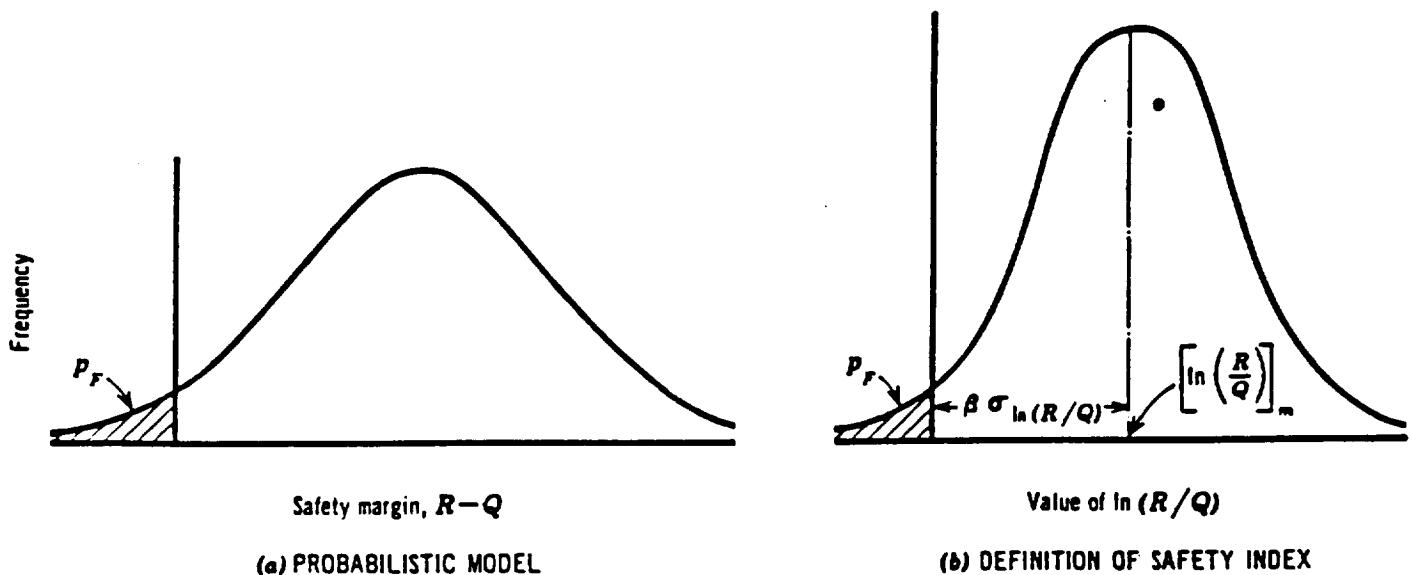
Figure 3-1: Definitions of Structural Safety [12]

If the "standardized variate" U is introduced, in which

$$U = \frac{\ln(R/Q) - [\ln(R/Q)]_m}{\sigma_{\ln(R/Q)}} \qquad (3\text{-}4)$$

where, $[\ln(R/Q)]_m$ and $\sigma_{\ln(R/Q)}$ are the mean and standard deviation of the natural logarithm of the ratio (R/Q), then from equation 3-3, the probability of failure can be written as given below

$$p_F = P\{U < -[\ln(R/Q)]_m/\sigma_{\ln(R/Q)}\}$$

$$= F_U\{-[\ln(R/Q)]_m/\sigma_{\ln(R/Q)}\} \qquad (3\text{-}5)$$

Here F_U is the cumulative distribution function of the standardized variate U. The quantity $[\ln(R/Q)]_m/\sigma_{\ln(R/Q)}$ defines the reliability of the element, thus it is called "safety index," β. If the probability distribution of (R/Q) were known, β would directly indicate a value of the probability of failure. In practice, the probability distribution of R/Q is unknown and only the first two statistical moments of R and Q are estimated. In the first-order probabilistic design method used here, β is only a relative measure of reliability; a constant value of β effectively fixes the reliability as a constant for all similar structural elements.

The expression for the **safety index** β,

$$\beta = [\ln(R/Q)]_m/\sigma_{\ln(R/Q)} \qquad (3\text{-}6)$$

can be simplified by using first-order probability theory into

$$\beta = [\ln(R_m/Q_m)]/\sqrt{(V_R^2+V_Q^2)} \qquad\qquad (3\text{-}7)$$

in which R_m and Q_m are the mean values of the resistance and the load effect, and V_r and $_QV$ are the corresponding coefficients of variation[12].

3.3 Load Combinations

Most load effects are random functions of time. The following are some important load combinations to be studied:

1. Dead load + lifetime maximum live load

2. Dead load + sustained live load +lifetime max wind load

3. Dead load + lifetime max live load + daily max wind load

4. Lifetime max wind load - dead load

5. Dead load + lifetime max snow load[13].

An examination of these loads follows.

3.3.1 *Live Load* - Statistical information on live loads is usually obtained from load surveys that give the live loads in the particular buildings surveyed at the times the surveys were made. From the load combination enumerated earlier, it is

seen that the distribution of the lifetime maximum live load is also needed. Pier and Cornell[15] have modeled the live load as consisting of the superposition of two parts: The sustained live load, which remains on the floor for a relatively long time until an occupancy change occurs, and the transient live load, which occurs infrequently but with a relatively high intensity and short duration. The sustained load includes furniture and normal working personnel. The transient live load may be caused by people in a room. Peir has proposed models to derive the statistics of the lifetime maximum sustained load and of the transient live load. Using the live load models of Pier and live load survey data of Mitchell and Woodgate[16], McGuire and Cornell[17] have derived the statistics of lifetime maximum live load.

3.3.2 **Wind Load.** – There are three random variables of interest in case of wind loads: The daily maximum, the annual maximum, and the lifetime maximum wind load. Meteorological data are available to derive the distributions of the daily maximum and of the annual maximum wind speeds throughout the United States. The lifetime maximum wind speed is approximately derived as the maximum of n-identically distributed and statistically independent random variables representing the annual maximum values, where n is the

lifetime of the structure in years. The mean and the coefficient of variation of the wind load(daily maximum, annual maximum, or lifetime maximum)are obtained taking into account the uncertainties in the dynamic characteristics of wind and the structure.

An analysis of 13 locations in the continental United States is given in the table 3-1 for a 1-yr period, which lists the location, the mean fastest mile daily wind speed, in miles per hour(V_{30Dm}), the corresponding 50-yr ANSI wind speed for the same location(V_{ANSI}), the factor $(V_{30Dm}/V_{ANSI})^2$ by which ANSI 50-yr wind pressure is multiplied to obtain the mean load intensity, and the coefficient of variation of the daily wind speed, V_{VD}[15].

TABLE 3-1 Maximum Daily Wind Statistics

Location	V_{30dm}, miles per hour	V_{ANSI}, miles per hour	$(V_{30Dm}/V_{ANSI})^2$	V_{VD}
Boston	21	90	0.05	0.32
Denver	19	80	0.06	0.38
Minneapolis	18	75	0.06	0.33
Chicago	18	80	0.05	0.30
St.Louis	18	70	0.07	0.37
Kansas City	18	70	0.06	0.39
Salt Lake C	18	80	0.05	0.39
Washington	17	75	0.05	0.36
Dallas	17	70	0.06	0.35
Atlanta	17	80	0.04	0.38
Pittsburgh	16	70	0.05	0.33
Seattle	16	80	0.04	0.37
New York C.	14	80	0.03	0.32

3.3.3 Load Factors- The purpose of load factors is to increase the loads to account for the uncertainties involved in estimating the magnitudes of dead or live loads. The usual load combinations to be considered are given below[19].

1. $U = 1.4D$

2. $U = 1.2D + 1.6L + 0.5(L_r$ or S or $R)$

3. $U = 1.2D + 1.6(L_r$ or S or $R) + (0.5L$ or $0.8W)$

4. $U = 1.2D + 1.3W + 0.5L + 0.5(L_r$ or S or $R)$

5. $U = 1.2D + 1.5E + (0.5L$ or $0.2S)$

6. $U = 0.9D - (1.3W$ or $1.5E)$

where, U = ultimate loads

D = Dead loads

L = Live loads

W = Wind loads

L_r = roof live loads

S = snow loads

R = rainwater or ice load

E = Earthquake load

3.4 Bending Resistance of Steel Beams

The plastic range represents the optimum capacity of the beam. Beams in this region are often described as compact beams[20]. In this range the plastic moment $M_p = F_y Z$ can be reached or exceeded, and this moment-level can be maintained for a large enough rotation so that inelastic force redistribution can take place and finally a mechanism can form. While in the elastic range of lateral-torsional buckling

the situation is clear, i.e., the member buckled or it is stable, the factors affecting the behavior in the plastic range are complex and intricately interrelated. Local flange, local web, and lateral-torsional distortions interact and they tend to build up gradually rather than form suddenly. Strain hardening on the one side and instability on the other side work against each other and they tend to balance out to give $M=M_p$ at the critical length L_p [21].

While much is known experimentally in the plastic range about the relationship of unbraced length and flange and web width-thickness ratios to rotation capacity, no generally satisfactory analysis that recognizes the complex interrelationships has yet been presented. Indeed, even if such a relationship did exist, its usefulness in design office situations would be questionable. Requiring designers to determine the required amount of rotation capacity to permit a desired level of moment redistribution would not be practical. The process is difficult, time consuming, and unreliable. Strain hardening significantly reduces the required rotation capacities based on ideal hinge behavior, i.e., $M_{max} = M_p$ [19].

Studies have been made on rotation capacity requirements of some general structures. These studies show that for

practical structures, the required rotation capacity is small(less than two). These are usually in extreme structures(single-story frames with very steep gables), or in zones of high moment gradient, where the ideal assumptions are invalid. In addition, these cases usually show that at a load just a few percent below the maximum, the requirements are greatly diminished. Current rules in plastic design are not based on any consistent rotation capacity requirements. The table below shows the statistical derivations of several tests on beams in plastic range.

TABLE 3-2 Statistical Data on Beam Tests in Plastic Range

Type of member	Number of tests	(Test/prediction)	V_p
Statically determinate beams under uniform moment	33	1.02	0.06
Statically determinate beams under moment gradient	43	1.24	0.10
Statically indeterminate beams and simple frames	41	1.06	0.07

3.5 Properties of Steel

The importance of material statistics may be, and is often, overshadowed by the uncertainties inherent in design. Required statistics of structural steel are not generally available for common grades of structural steel because steel specifications and material specifications work with specified minimum values. Examining the existing literature on material properties of structural steel is, therefore, necessary and to obtain an estimate of the properties needed. Characteristic and representative sets of data were examined and estimates were made of the mean values and the coefficients of variation for tentative use. The principal material property affecting the resistance of a steel structure is the yield stress[22]. The values for use as proposed by T.V.Galambos et al is given in the table 3-3 overleaf.

TABLE 3-3 Summary of Material Properties Used in LRFD

Criteria

Material Property	Mean Value, in kips per square inch	Coefficient of variation
Modulus of elasticity in tension	29000	0.06
Modulus of elasticity in compression	29000	0.06
Modulus of elasticity in shear	11200	0.06
Poisson's ratio	0.30	0.03
Yield stress in flanges	1.05 F_y	0.10
Yield stress in webs	1.10 F_y	0.11
Yield stress in shear	0.64 F_y	0.10

3.6 Variation of Safety Index

The value of safety index may be varied to account for the importance of the structure. If the structure is important(like public buildings, national monuments, places of worship, industries etc.), then they can be designed for a higher reliability factor, to take care of the stochasticity

of loading in these buildings. There are some structures in which failure of one or a few critical elements may result in the total loss of the structure ("weakest link" type structures) in contrast to ductile or continuous structures ("parallel" type structures).

Optimal levels of reliability for different types of structures could be obtained from an expected total cost optimization process. It could be decided, that $\beta=3.0$ in ordinary buildings, $\beta=4.5$ for very important buildings, and $\beta=2.5$ for temporary structures. It is possible to incorporate the statistical correlation between cross sections and between members and failure modes by suitably varying the value of the safety index β [10]. The LRFD formulation is versatile enough to incorporate these future developments in probabilistic design.

3.7 Comments on LRFD codes

The simple structures used by the LRFD approach to calibrate the load and resistance factors have closed form solutions, i.e., the response in these structures is available as an analytical expression in terms of the basic structural parameters. Therefore the limit state is also analytically available, making it easy to estimate reliability. However,

for most realistic structures the response is not **available** as a closed-form solution; it can only be evaluated through numerical procedures such as finite element analysis. Therefore, more complicated numerical procedures than those used in LRFD are needed to estimate the reliability of members in such structures.

In the LRFD approach, however, the individual members in complicated structures are designed using the same load and resistance factors that were derived based upon the reliability analysis of simple structures. The use of isolated simple structures to derive safety factors is related to the basic design philosophy common to all codified design procedures. There are several advantages to the isolated-member approach: (1) In deterministic design methods that use factors of safety, preparing detailed requirements for each structural configuration is not practical; (2) the characteristics of the individual members and connections themselves are independent of the framework; and finally, (3) most research has been devoted to the study of such elements, and theoretical and experimental verification of their performance is readily available. Nevertheless, the performance of a member is directly dependent on its location in a structural configuration and its relationship or

connection with other members in the framework. Such dependence is not restricted to the computation of load effects through a deterministic analysis of the structure, but extends to the probabilistic characteristics of all the parameters of the structure. Only a probabilistic structural analysis of the entire structure can account for such influence and accordingly determine the risk or reliability of any individual member, thus enabling an improved approach to reliability-based design.

An important objective of the reliability-based design methods such as LRFD is to reduce the scatter of nonuniform risk levels produced under various load combinations by the conventional design methods. As described in the AISC LRFD specification (Manual 1986), the reliability indices inherent in the 1978 AISC specification (Manual 1978), when evaluated under different load combinations and for various tributary areas of typical members, show a considerable range of variation. The LRFD approach seeks to narrow this range of variation of β values by specifying several "target" β values and selecting multiple load and resistance factors to meet these targets. Since the computation of β values in this approach is based on direct simulation using simple, isolated structural elements, an improved analysis of a realistic

structure would reveal that, even for the same load combination and the same limit state, there is considerable variation in the β values among the different members of the structure. Thus, there is further scope for improvement in the achievement of uniform risk by reducing the variation of β values among different members in a structure, within the limitations of practical design. This is also advantageous from the point of view of structural optimization as in weight minimization, since uniform risk among members implies a balanced distribution of weight.

A third aspect of the LRFD approach, which needs closer examination and possible improvement, is the consideration of the statistical correlation among the basic structural variables. The load and resistance factors in the LRFD approach were derived assuming statistical independence of variables. This may be reasonable for isolated simple members, which do not have too many variables, and assumption of lack of correlation may not significantly affect the determination of the reliability index. However, for members in structures such as frames, correlations among the random variables may have a significant effect, and so need investigation.

The key to successful resolution of all these issues is the ability to perform reliability analysis of complicated

structures for which the response is not available as a closed-form solution in terms of the input variables, except in an algorithmic form such as finite-element code, like "NESSUS."

CHAPTER IV

SYSTEM RELIABILITY ANALYSIS OF STEEL FRAMES

4.1 Reliability Analysis of Complicated Structures

Three types of solution strategies are possible for the reliability analysis of complicated structures; they are: (1)Direct simulation (2)approximation of the performance function by a polynomial; and (3)the stochastic finite element method[4].

The stochastic finite element method uses a more direct approach to the reliability analysis of structures. Starting with second-order statistics of the basic random variables, it keeps account of the variation of the quantities computed at every step of the deterministic analysis with respect to the basic random variables, and thus makes it possible to compute the statistics of response or the reliability for any limit state.

For structures whose limit state is not available in closed form, Wu(1984) suggested the use of a simple, easily constructed second-degree polynomial that approximates the limit state in the neighborhood of the design point. Repeated deterministic analysis at selected points in the neighborhood and subsequent regression analysis are used to achieve this

objective. Then the Rackwitz-Fiessler algorithm is used to estimate the reliability index, through the solution of the approximate limit-state equation.

Direct simulation, though robust is expensive. A large number of deterministic runs are required to compute the probability of failure, which is generally required to be very low in conventional civil engineering structures. The efficiency of the simulation can be improved by reducing the variance of the estimated probability of failure, which uses the same execution times and storage requirements without disturbing its expected value. Several such variance reduction techniques have been proposed and used in structural reliability analysis, e.g., importance sampling method. These variance-reduction techniques can also be combined further to increase the efficiency of the simulation.

This chapter develops a method to quantify the effect of different types of collapse mechanisms of a structure under cumulative loading with the help of numerical examples. The purpose of the numerical examples is twofold: First, to illustrate reliability analysis of steel frames for the performance functions presented later in this chapter; and second, to examine steel frames designed using the LRFD approach and determine whether the target reliabilities of the

structures have been attained considering the overall structural configuration.

4.2 Statistical Information

For the reliability analysis, a probabilistic description of the variables is necessary. The stochastic variation of loads, material properties, sectional properties have been extensively studied in the earlier chapter; according to existing literature. Ellingwood et al.(1980) provided detailed statistical information, including the type of distribution, about some of these parameters.

The dead load and all the resistance variables have been described as lognormal variables; wind load and the live load were described as type I extreme value variables.

4.3 Statement of the Numerical Example

Examine several steel structures, designed using the LRFD approach, without changing the structural configuration, but by varying the structural geometry and the loads. Determine whether the target system reliabilities, as stated by the codes are reached. Interpret the results obtained.

4.4 Analysis of the Problem

One basic structural configuration for the plane steel frame is chosen. Using the same structural configuration, the structural geometry and the loads were varied to give sixteen different structures were obtained for analysis. The nominal values of the dead loads, live loads, wind loads are calculated to the best possible alternative, with the help of UBC codes[22](1988). These values are tabulated for different runs, in table 4-1.

4.4.1 Assumptions in analysis

The following assumptions were made in the analysis:

1. Elasto-plastic framed structures are used. If a moment exceeds the moment capacity at a section, a plastic hinge occurs and an artificial moment of magnitude equal to its resistant moment capacity is imposed at this section. Component failure due to buckling and violation of displacement constraints is not considered.

2. The structural uncertainties are represented by considering only the moment capacities as random variables.

3. Geometrical second-order and shear effects are neglected. The effect of axial forces on the reduction of moment capacities are also neglected.

4. The order of loads and loading paths are not considered.

These assumptions are often used in time-invariant system reliability analyses for ductile frame structures.

4.5 Applied Plastic Design

Until recent years most steel beams were designed based on elastic theory. The maximum load that a structure could support was assumed to equal the load that first caused a stress somewhere in the structure to equal the yield stress of the material. The members were designed so that the computed bending stresses for service loads did not exceed the design stress. Engineering structures have been designed for many decades by this method with satisfactory results. The design profession, however, has long been aware that ductile members do not fail until a great deal of yielding occurs after the yield stress is first reached. This means that such members have greater margins of safety against collapse than the elastic theory seems to indicate.

This sums up the basis of the plastic theory. The theory is that those parts of the structure stressed to the yield point cannot resist additional stresses. They instead will yield the amount required to permit the extra load or stresses

to be transferred to the other parts of the structure where the stresses are below the yield stress and thus in elastic range and able to resist increased stresses. Plasticity can be said to serve the purpose of equalizing stresses in cases of an overload.

A statically determinate beam will fail if one plastic hinge develops. For a statically indeterminate structure to fail it is necessary for more than one plastic hinge to form. The number of plastic hinges required for failure of statically indeterminate structures will be shown to vary from structure to structure, but may never be less than two.

One very satisfactory method used for plastic analysis is the virtual-work method. The structure in question is assumed to be loaded to its nominal capacity, Mn, and is then assumed to deflect through a small additional displacement after the ultimate load is reached. The work performed by external loads during this displacement is equated to the internal work absorbed by the hinges. For this discussion the small-angle theory is used. By this theory the sine of a small angle equals the tangent of that angle and equals the same angle expressed in radians. We can use these values interchangeably because the small displacements produce extremely small rotations or angles[19].

This theory is the basis, of formulation of limit state functions in the analysis of the structures under consideration in the problem.

4.6 Design of Structures

A method is proposed to design the structures, in which, the objective is to estimate the element reliability under material nonlinearity represented by plastic hinge and not system reliability. The emphasis is on identifying the important linear segments of the nonlinear element reliability limit state through this procedure. In terms of implementation, the proposed method imposes a group of plastic hinges on the structure, instead of imposing only one hinge at each step as in current system reliability methods, as developed by Xiao, et al[7]. This grouped imposition is an important step that saves much computational effort for large structures since the number of structural reanalyses is greatly reduced. Particular group of plastic hinges, which will produce significant change, is isolated.

4.6.1 Stepwise Design Procedure

The algorithmic design procedure can be clearly seen in the flow chart as seen in Figure 4-1.

Start

Set up geometry of the structure.
Control the slenderness ratio by controlling height.
No buckling.

Assume tributary area and estimate loads according to Uniform Bldg. Codes.

Vary the loads and the geometry of the structure to obtain 16 different configurations.

Conduct a force study of the structures using any structural FEM software.
Use all critical LRFD loading combinations, as given in the code.

Isolate sections of maximum moment to conduct a moment design.

Use hand calculations/computer program to calculate and design all members in the structure, using the LRFD model formula as developed by Cornell.

Carry out indeterminacy study to find out the number of limit states in the structure.

Isolate critical failure modes/ mechanisms by branch and bound method.

Formulate response function for each mode using applied plastic design and principle of virtual work.

Assign COV and probability distributions to random variables, as per statistics of the LRFD code.

Perform fault tree analysis and calculate system safety index for all 16 cases, using NESSUS.

Study the variation of the safety index.

Is the safety index below target?   YES

NO

The design is unsafe.

NO

Is the safety index considerably higher than target ?

YES

System design by LRFD is conservative.

System design by LRFD is O.K.

LRFD criteria is doubtful for system design.

STOP

Figure 4-1 : Flow Chart for Design Procedure

The following steps were incorporated to design the structures under consideration:

Step 1 - All the initial nominal values of the dead load D, live load L, and wind load W, are selected based on the UBC building codes, taking Nashville, TN, as the location center. The calculations* yielded the nominal values given in the table below.

TABLE 4-1   NOMINAL VALUES OF LOADS

Variable	Nominal Value
D - Dead Load	4.0 kips
L - Live Load	7.5 kips
W - Wind Load	2.0 kips

The basic structure to be analyzed is seen in Figure 4-2, with dimensions and loading patterns. The live load and the dead load are applied at the center of each beam, with the wind load point application at the node of the column-beam junction. Based on this basic structural configuration, fifteen variations physically possible with variations in wind load,

Figure 4-2 : Basic Structural Configuration under analysis
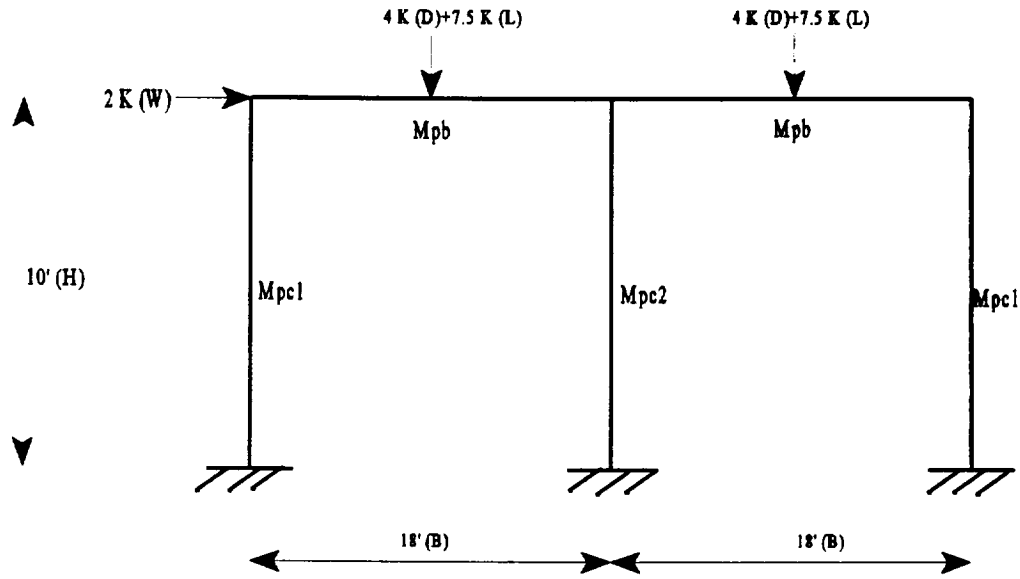
vertical dead and live load, horizontal bay
dimensions and vertical height dimensions, were
thought of; thus making a total of sixteen
structures to be analyzed by the proposed method.

The values of the wind load, live load, dead
load, bay dimension, height dimensions are given in
table 4-2 below for all 16 cases.

TABLE 4-2   VARIATIONS OF NOMINAL VALUES

Case #	Dead load (D); kips	Live load (L); kips	Wind load (W); kips	Bay size (B), feet	Height (H), feet
Case 1	4.0	7.5	**2.0**	18.0	10.0
Case 2	4.0	7.5	**4.0**	18.0	10.0
Case 3	4.0	7.5	**6.0**	18.0	10.0
Case 4	4.0	7.5	**8.0**	18.0	10.0
Case 5	**6.0**	**10.0**	2.0	18.0	10.0
Case 6	**8.0**	**12.5**	2.0	18.0	10.0
Case 7	**10.0**	**15.0**	2.0	18.0	10.0
Case 8	**12.0**	**17.5**	2.0	18.0	10.0
Case 9	4.0	7.5	2.0	18.0	**12.0**
Case 10	4.0	7.5	2.0	18.0	**14.0**
Case 11	4.0	7.5	2.0	18.0	**16.0**
Case 12	4.0	7.5	2.0	18.0	**18.0**
Case 13	4.0	7.5	2.0	**20.0**	10.0
Case 14	4.0	7.5	2.0	**22.0**	10.0
Case 15	4.0	7.5	2.0	**24.0**	10.0
Case 16	4.0	7.5	2.0	**26.0**	10.0

Step 2 -   The next step was to conduct the force study of all
             the 16 different structures. This was done with the
             aid of a structural finite element software, STAAD -

III, developed by Research Engineers Inc. The
loading cases analyzed in this study according to
LRFD[2] formulae were :

$$U = 1.4D \qquad\qquad \text{--- LRFD A4-1 (4-1)}$$

$$U = 1.2D + 1.6L \qquad \text{--- LRFD A4-2 (4-2)}$$

$$U = 1.2D + 0.5L + 1.3W \quad \text{--- LRFD A4-4 (4-3)}$$

The outputs of the individual member forces were
studied, with moment in the Z-direction being the
prime governing factor as discussed in the
assumption of analysis of the problem. The sections
of maximum moment isolated, ford, for use in the next
step of the design process that would be plastic
design based on LRFD codes to design each member of
all the 16 structures.

Step 3 - A computer program* was developed on the lognormal
distribution for β, as is discussed in Chapter III.
The formula follows:

$$\beta = [\ln(R_m/Q_m)]/\sqrt{(V_R^2 + V_Q^2)} \qquad\qquad (4\text{-}4)$$

Where,

R_m = the mean resistance

Q_m = the mean load effects,

which in our design process would be the plastic moment of the beam (including the effect of ϕ) and the maximum moment induced in the beam derived from the force study. The target β and ϕ used for the columns and beams are given in the table 4-3 below:

TABLE 4-3   TARGET VALUES OF β AND ϕ FOR ELEMENTS

	β	ϕ
COLUMNS	2.5	0.85
BEAMS	3.0	0.90

V_r and V_Q are the coefficients of variation of the plastic moment of the beam and the moment induced. Both of these coefficients of variations are numerically used as 0.10 as discussed in Chapter III.

After using the program developed, all structures were designed for element reliabilities desired. The results of the design are tabulated in table 4-4.

* listed in APPENDIX 'A'

TABLE 4-4   DESIGN SECTIONS

CASE #	BEAM SECTION (B)	EXTERNAL COLUMN SECTION (C1)	INTERNAL COLUMN SECTION (C2)
1	W8X28	W8X10	W6X9
2	W8X28	W8X10	W6X12
3	W8X28	W8X15	W8X15
4	W8X28	W8X18	W10X15
5	W10X33	W10X12	W6X9
6	W12X35	W10X15	W6X9
7	W10X49	W6X25	W6X9
8	W12X50	W8X24	W6X9
9	W10X22	W6X12	W6X9
10	W12X22	W6X12	W6X9
11	W12X22	W6X12	W4X13
12	W12X22	W8X15	W6X12
13	W8X31	W6X15	W6X9
14	W14X22	W6X16	W6X9
15	W10X30	W8X15	W6X9
16	W10X33	W10X15	W6X9

The W-sections chosen for design are in accordance with the LRFD, AISC specifications. It should be noted that some sections are not practically

available in the market, but are listed in the codes. They are solely selected for theoretical purposes of close simulation to design conditions.

Step 4 - A complete indeterminacy study was carried out of the structures. The following results were deduced:

|	
Number of possible hinges formed	= 10
Number of redundancies	= 6
--	
Number of independent mechanisms	= 4

Out of these 4 failure mechanisms, 3 critical mechanisms are identified-(1) beam mechanism; (2) column mechanism;(3) combined mechanism.

Step 5 - The next step was the formulation of the 3 g-functions based on the virtual work study[23] of all the 3 mechanisms. The rotations, at the end of the plastic hinges are equal. The Figure 4-3 shows the different modes of failure possible in the structure.
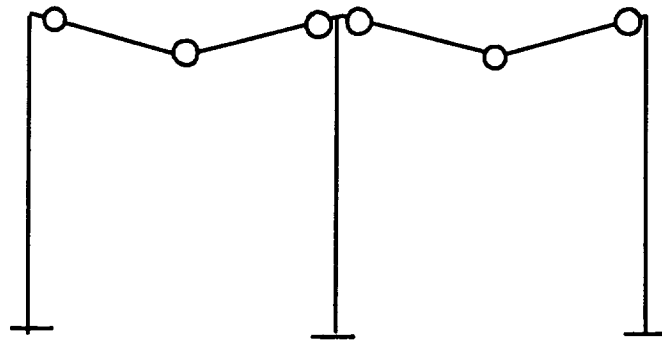
Mode 1 - Beam Mechanism
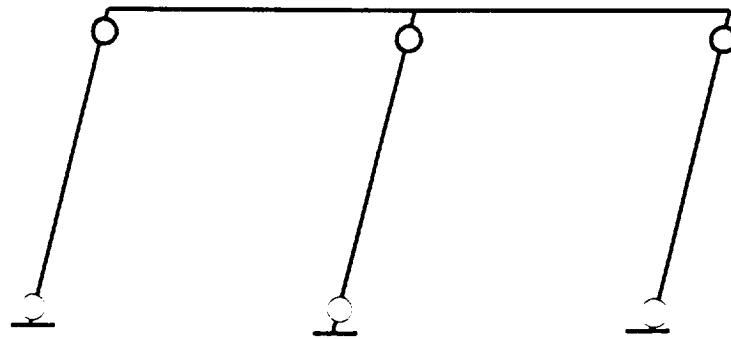
Mode 2 - Column Mechanism
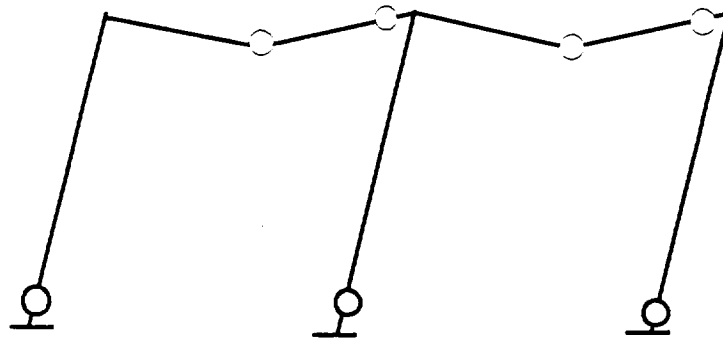
Mode 3 - Combined Mechanism

Note : All circles indicate the formation and location of possile plastic hinges.

Figure 4-3 : Significant Modes of Failure in structure.

The external work performed is **always** the product of the load and the average deflection of the mechanism. The average deflection equals one-half the deflection at the center or dominating plastic hinge. The internal work is the sum of M_p at each plastic hinge times the angle through which it works. The resulting expressions for the g-functions which is a result of difference between the internal resistance(work) and the external work performed. They are formulated below:

1. BEAM MECHANISM:

$$g_1 = 8.0*M_{pb} - (D+L)*B$$

2. COLUMN MECHANISM:

$$g_2 = (4.0*M_{pC1} + 2.0*M_{pC2}) - W*H$$

3. COMBINED MECHANISM:

$$g_3 = (2.0*M_{pC1} + 3.0*M_{pC2} + 12.0*M_{pb}) - (W*H + B*(D+L))$$

Step 6 - This step is explicitly explained in Chapter V, which concentrates on the formation of the fault tree risk analysis formation and discussion of the results.

CHAPTER V

SYSTEMS RISK ANALYSIS AND RESULTS

System risk analysis is carried out using "NESSUS" by the development of fault trees that combine different modes of failure in the system.

5.1   Fault Tree Analysis

In calculating system reliability, it is important to include the probabilistic dependencies between multiple component failures, or between different failure modes. Failure to do so could result in significant errors. Fault tree analysis is a commonly used tool in risk assessment. A Fault tree is a mathematical construction of assumed component failure modes (bottom events) linked in series or parallel leading to a top event, which denotes the total system failure. A fault tree diagram essentially decomposes the main failure event (top event) into unions and intersections of subevents or combination of subevents. The decomposition continues until the probabilities of the subevents can be evaluated as single mode failure probabilities. The probabilistic fault-tree analysis is based on the limit state definition of the bottom events. Thus, one requirement for

system risk assessment is to compute failure function of each bottom event. Each bottom event is defined by a close form equation.

A fault-tree has three major characteristics: bottom events, combination gates and the connectivity between the bottom events and the gates. The system risk assessment is limited to AND and OR gates. The OR gate implies that the output fault event is the union of subevents. The AND gate signifies that the output fault event is the intersection of the subevents. The different steps involved in the application of the fault-tree analysis method can be summarized as follows[24].

1. Development of a fault tree to represent the structural system.

2. Construction of an approximate performance function for each bottom event.

3. Determination of a dominant sampling sequence for all bottom events.

4. Calculation of the system reliability using Adaptive Importance Sampling method.

To illustrate the Fault-tree analysis, consider a simple example consisting of two failure modes: yielding and

excessive displacement. Two failure functions can be expressed as,

$$g_1 = R \text{ (Yield strength)} - S \text{ (Stress)} \qquad (5\text{-}1)$$

$$g_2 = D \text{ (Allowable displacement)} - d \text{ (displacement)} \qquad (5\text{-}2)$$

Failure occurs if $[g_1 < 0]$ or $[g < 0]$. Using standard probability notations, the system probability of failure is:

$$P_f = P[(g_1 < 0) \cup (g_2 < 0)] \qquad (5\text{-}3)$$

In general,

$$P_f = P[g_1 < 0] + P[g_2 < 0] - P[(g_1 < 0) \cap (g_2 < 0)] = P_1 + P_2 - P_{12} \qquad (5\text{-}4)$$

In general, P_{12} ranges from 0 to the smaller value of P_1 and P_2 therefore, P_f ranges from $[P_1 + P_2]$ to P_2 (assuming $P_2 > P_1$). Hence, by assuming independent events, the error ranges from $-P_1 P_2$ to $P_1 (1 - P_2)$.

In application to the project, one OR gate is considered with three bottom events. The three bottom events represent the three failure modes of the structure. The representation of Fault-tree with three failure modes is shown in Figure 5-1.
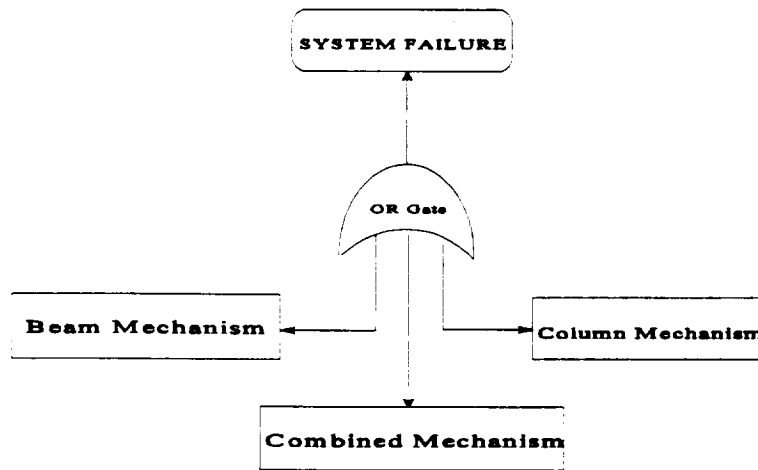
Figure 5-1 : Representation of fault tree analysis

The Fault-tree analysis is carried out by two methods. They are:

    1. Adaptive importance sampling method.

    2. Standard Monte Carlo sampling method.

5.1.1 Adaptive Importance sampling method

Adaptive Importance Sampling is different from traditional importance sampling methods because of its ability to adjust automatically and by that reduce the sampling space. Because of this attribute, adaptive importance sampling method is highly efficient and accurate alternative for probabilistic analysis.

Two options are available for selecting the sampling

boundaries. The first order adaptive sampling method uses hyperplanes, and the second-order adaptive sampling method uses parabolic surfaces. Both surfaces are constructed in the u-space and use the most probable point to define the beginning sample space. In general sampling space can be adjusted by increasing or decreasing the curvatures of the parabolic surface until there are no more failure points in the final sampling space. In the first order-based method, only the distance to the hyperplane is changed. In the second-order-based method, the curvature of the sampling boundary is updated first, then the final surface is shifted toward the origin[12].

5.1.2 Monte Carlo Sampling method

Monte Carlo sampling method is a way of generating information for a simulation when events occur in a random way. It uses unrestricted random sampling (it selects items from a population so that each item in the population has an equal probability of being selected) in a computer simulation in which the results are run off repeatedly to develop statistically reliable answers. A sample from a Monte Carlo simulation is similar to a sample of experimental observations. Therefore, the results of Monte Carlo

simulations may be treated statistically. Monte Carlo methods are useful because they can handle very complex models, are guaranteed to work, and are exact in the limit as the number of samples becomes large. The disadvantage is that a very large number of simulations may be necessary[25].

5.2 Structural System Reliability using NESSUS

System reliability considers failure at multiple locations, multiple failure modes, multiple components and combinations of all three. System reliability in NESSUS is currently addressed by a probabilistic fault tree analysis(PFTA) method. The driver module for system reliability is the SRA module with the PFTA methodology in the FPI module. The procedure implemented is intended to be accurate and efficient and build off the previous capabilities of NESSUS.

The user defines system failure through the fault tree by defining the bottom events and their combination with "AND" and "OR" gates. Each bottom event considers a single failure, i.e., component reliability, and is defined through a finite element model and performance function. NESSUS will compute the reliability of each bottom event and a polynomial approximation, called a failure function, to the structural

response at the most probable point(MPP) using the AMV+ algorithm. The failure functions are then combined according to the fault tree[26]. System reliability is then computed using an adaptive importance sampling method. The adaptive importance sampling in this module has two features. First, the sampling region is focussed on the most important region where it has the highest probability of failure, and second, the sampling region is not predetermined. Instead, the sampling region is gradually increased by deforming the sampling boundary until the sampling region fully covers the failure region sufficiently. When the sampling region fully covers the failure region, the probability solution will converge, indicating that no more deformation is required[27].

There are several advantages to this approach. Because the failure functions are used, not just the probability of failure of each bottom event, the method can account for correlation between bottom events. The preexisting NESSUS capabilities for component reliability and failure function for each bottom event. In addition, adaptive importance sampling is typically an order or more faster than conventional Monte Carlo.

The PFTA procedure implemented in NESSUS is being investigated for use with progressive fracture failure mode.

5.3 RESULTS OF SYSTEM RELIABILITY CALCULATIONS

The PFTA method uses the failure function about the MPP for each bottom event not just the probabilities.

A summary of the system probalilities of failure and the respective safety indices for all the 16 cases are given in the table below

TABLE 5-1   PROBABILITY AND SAFETY INDEX RESULTS

Case #	Probability of Failure (P_f)	Safety Index (β)
Case 1	0.15728E-07	5.534
Case 2	0.14167E-03	3.630
Case 3	0.83901E-04	3.763
Case 4	0.21916E-03	3.516
Case 5	0.56805E-10	6.448
Case 6	0.54563E-12	7.118
Case 7	0.27913E-12	7.210
Case 8	0.67462E-13	7.401
Case 9	0.86963E-06	4.782
Case 10	0.85838E-05	4.299
Case 11	0.47044E-04	3.905
Case 12	0.51779E-06	4.885
Case 13	0.97542E-09	6.002
Case 14	0.21091E-09	6.246

Case 15	0.15191E-10	6.645
Case 16	0.22340E-11	6.922

Considering the results, which we can even see in a graphical form as seen in figure 5-2, there is clearly a difference between the safety indices of the four different variations tried in the structural loadings and geometries, and we see that the system safety index is consistently higher than the target. This can be seen in the table 5-2 below;

TABLE 5-2   SAFETY INDEX VARIATIONS

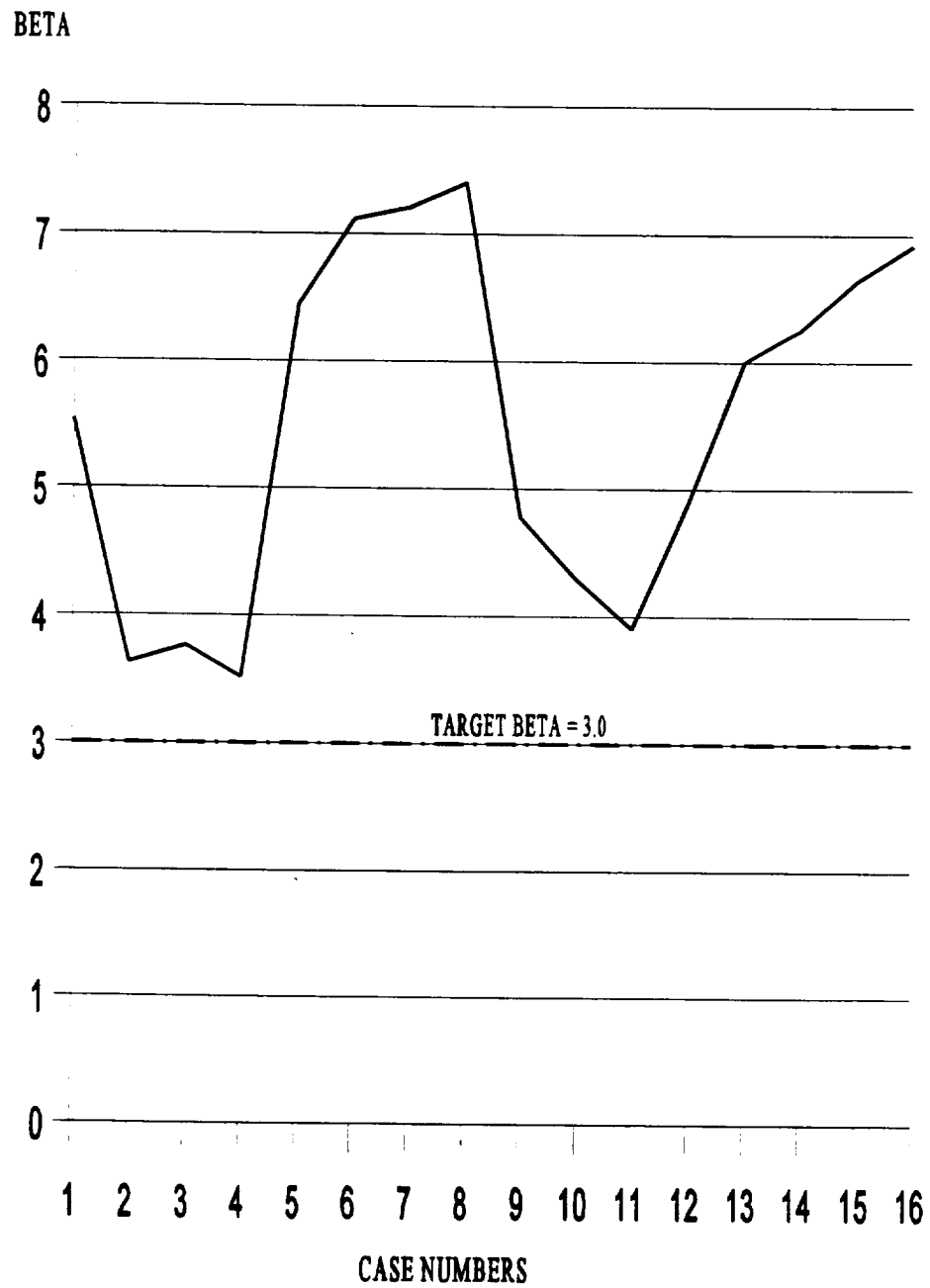| CASE #	AVERAGE SAFETY INDEX (β_{AVG})	TARGET RELIABILITY INDEX (β_{TARGET})
CASE 1 - CASE 4	4.111	3.0
CASE 5 - CASE 8	7.044	3.0
CASE 9 - CASE 12	4.468	3.0
CASE 13 - CASE 16	6.454	3.0

BETA

Figure 5-2 : Safety Index variation in comparision with target safety index

These results are obtained by using the same LRFD criteria used for developing the codes, using first order probabilistic design. After these results were obtained, a sensitivity analysis was done for the first case, taking it as a representative structural configuration. The results of which are graphically represented in fig 5-3 to fig 5-5. From these graphs, we conclude that the live load is the most sensitive variable in the beam collapse mechanism and the combined collapse mechanism, and the plastic moment of the external columns is the most sensitive variable in the column collapse mechanism.

It is decided to vary, the coefficients of variations of the vertical live load, plastic moment of the external column section, horizontal wind load, beam section; to study the effect of these variations of individual limit state variables on the safety index of the system. The results are depicted in fig 5-6 to fig 5-9. It is worth noting from fig 5-6, that even though the live load is the most sensitive parameter for two limit states, it has no effect at all on the system reliability. However, the plastic moment of the external columns, which is the most sensitive parameter for one limit state, affects the system reliability(safety index) considerably with change in its statistics. This means that

the plastic moment of the external column is the most critical parameter in the system. It also proves that even though a particular variable is sensitive in a single limit state, it may not be the most critical when system effects are accounted.

5.4 Observations

The main objective of this project - the validation of LRFD for actual structures - is achieved by comparing the reliability indices(computed using NESSUS) for the various limit states in a plane frame structure, with the target values used in LRFD. As seen in tables tabulate the NESSUS-computed β values for the structures and also the target β values show that the former is consistently higher.
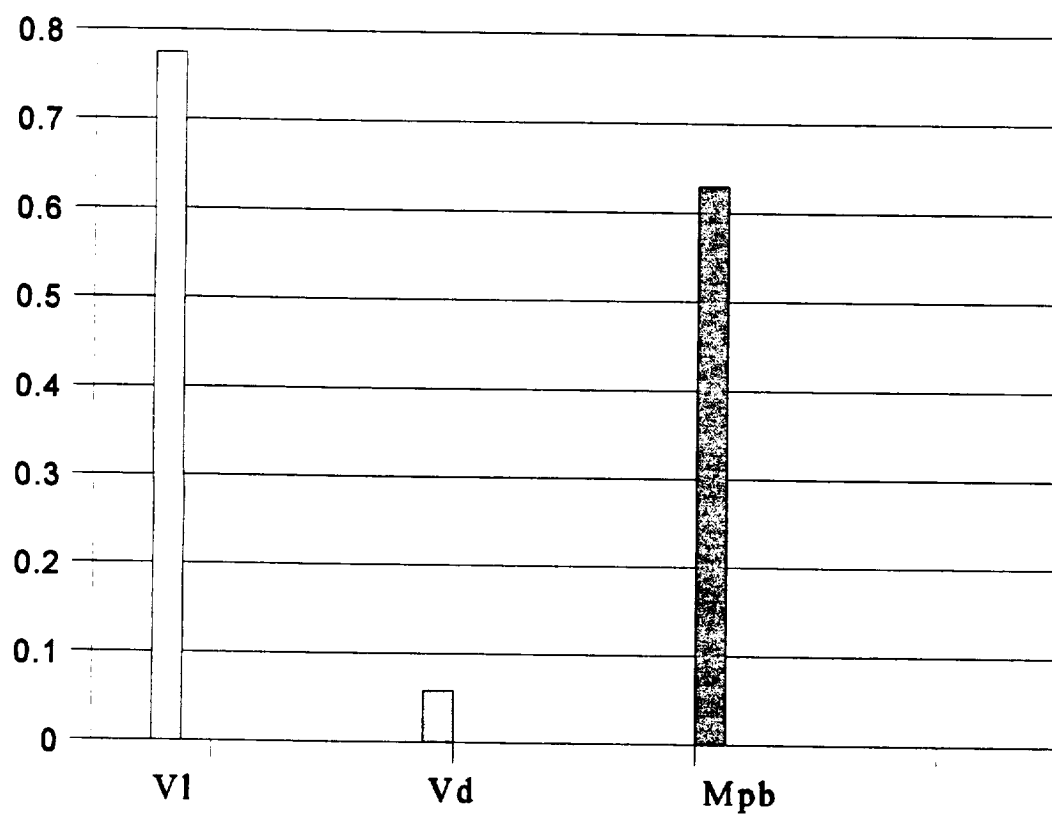
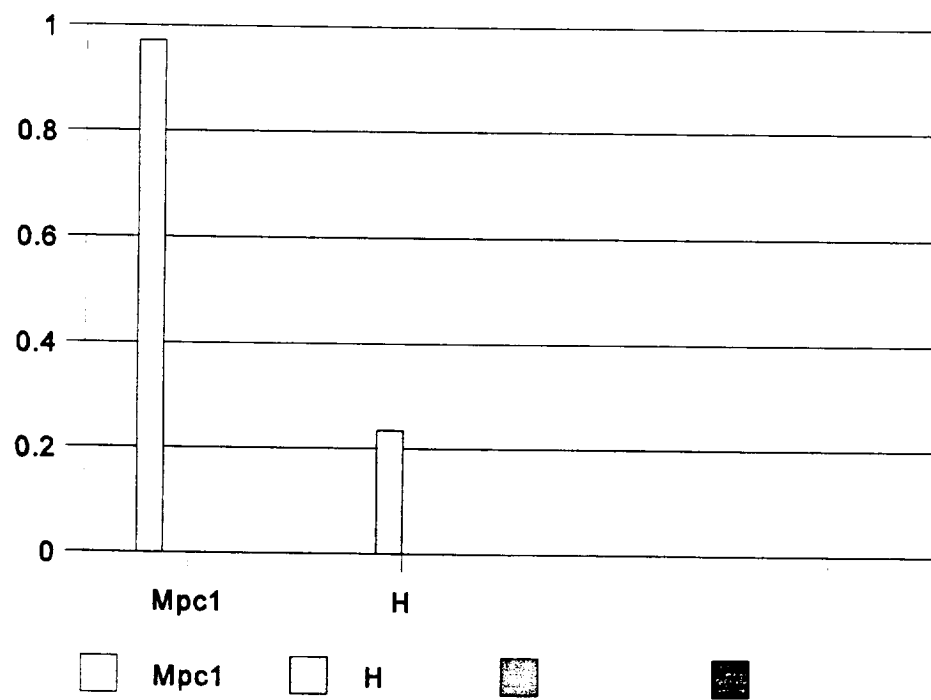FIGURE 5-3 : Beam Mechanism Sensitivity Analysis

FIGURE 5-4 : Column Mechanism Sensitivity Analysis
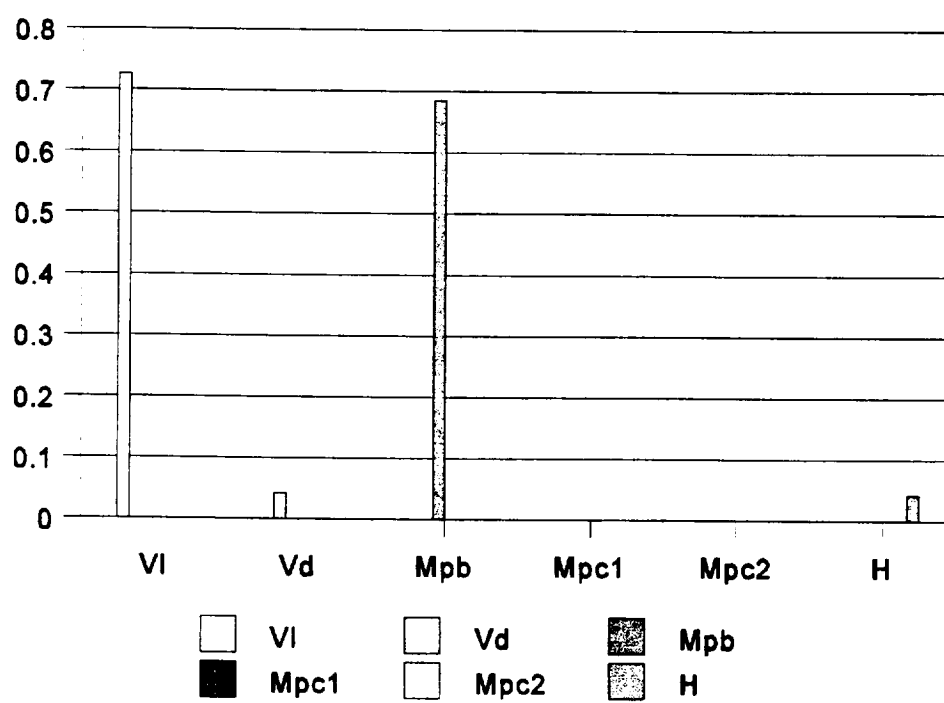
FIGURE 5-5 : Combined Mechanism Sensitivity Analysis

Figure 5-6 : Relationship between Beta and Coefficient of Variation
With variation in live load.

Figure 5-7 : Relationship between Beta and Coefficient of Variation
With variation in column section.
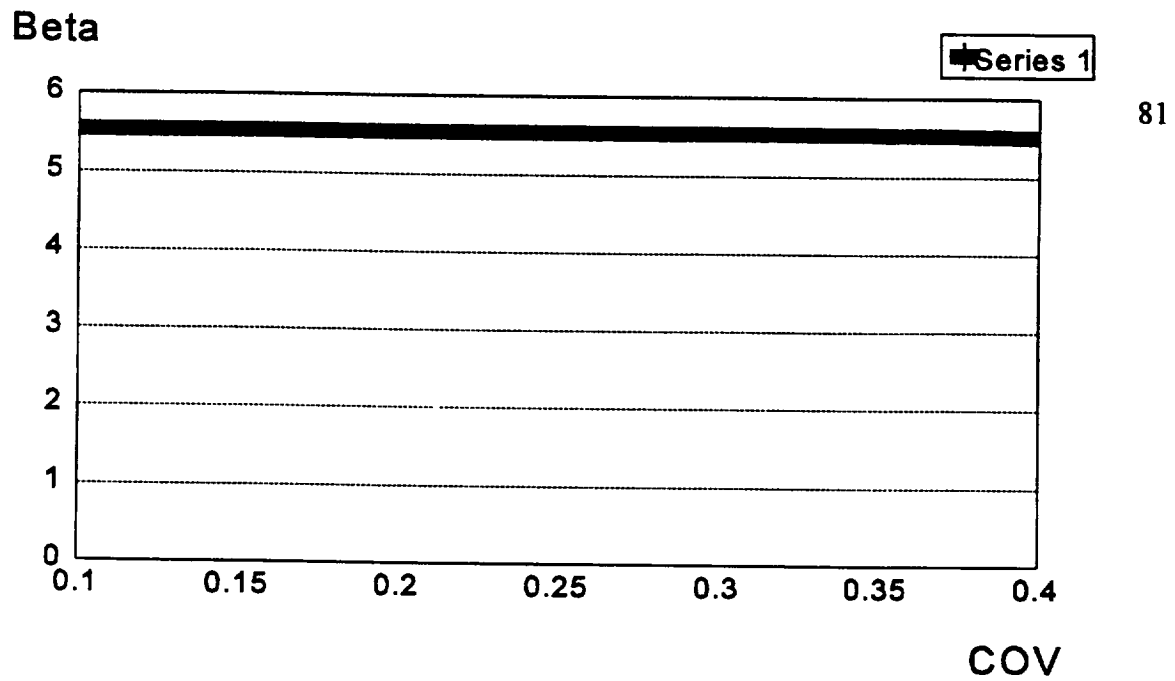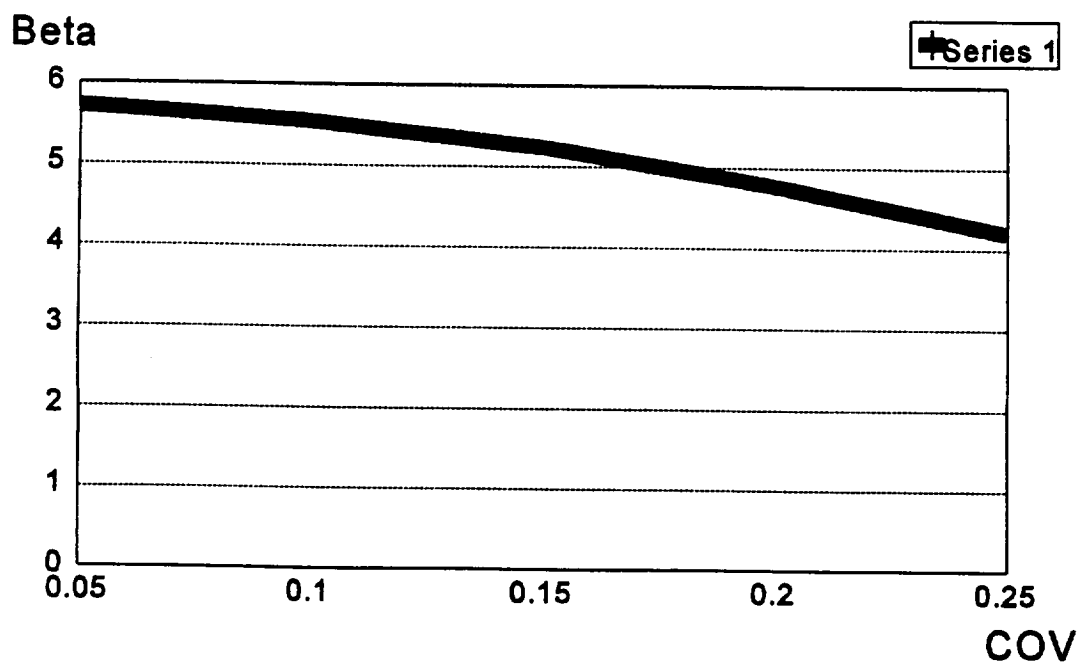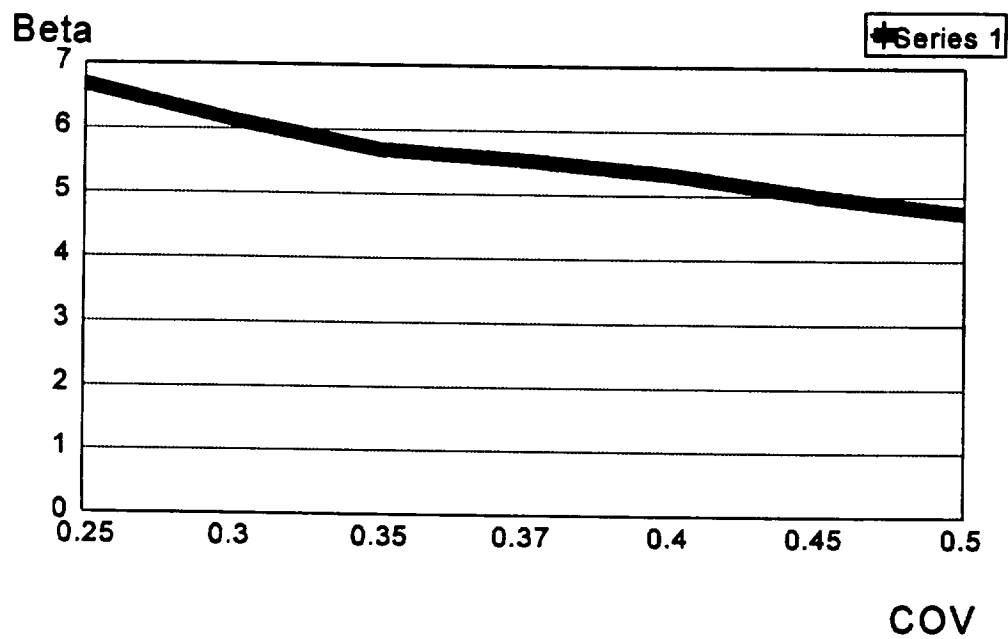
Figure 5-8 : Relationship between Beta and Coefficient of Variation
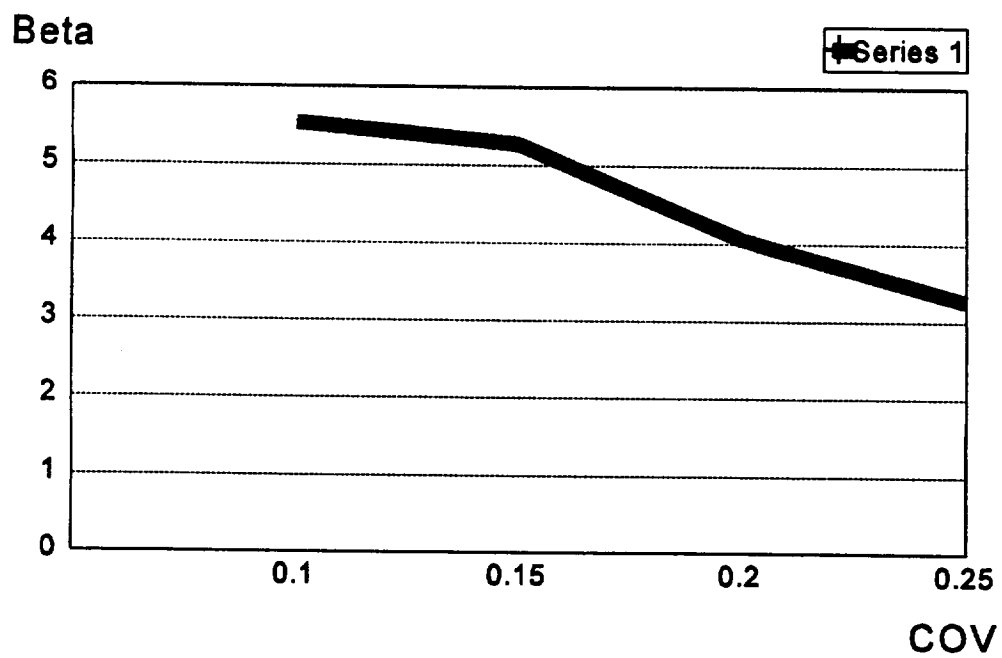With variation in wind load.

Figure 5-9 : Relationship between Beta and Coefficient of Variation
In Beam Section.

In cases 1-4, where, the design is determined by the variation of horizontal wind load, the β_{AVG} is 4.111. In cases 5-8 the design is dominated by the variations in vertical load, the β_{AVG} is 7.044, which is more than twice the target β. The dominating factor in the design decision is variations in height of the structure, here the β_{AVG} is 4.468. Finally the rest of the cases are dominated by change in bay dimensions of the structure, where the β_{AVG} is 6.454.

The NESSUS approach used here to estimate the system reliabilities designed according to LRFD differs from the latter in two respects: The effect of all structural variables is considered while estimating the reliability index for any particular case of the structural configuration whereas the LRFD method deals with the reliability of isolated members. In the approach described in this project, correlations are assumed between some random variables. Apparently, the load and resistance factors used by the LRFD approach are conservative, resulting in higher reliability of structures than the target reliabilities. The use of "standard" design situations such as simple beams, centrally loaded columns, tensile members, etc. to derive the load and resistance factors appears to have resulted in a conservative design for more complicated situations such as frames.

There is no appreciable effect of correlations between sectional properties on the reliability index. This is not surprising, since sectional properties show very little random variation(coefficient of variation is 0.1), whereas the largest variations are in the loading variables. Therefore, if there are any correlations among the load variables, it is possible that these might be more significant.

If the proposed method, can give system structural reliability results to certain degree of accuracy, it is possible to use this method to formulate a procedure or relationship for optimum structural strength, ensuring uniform risk among different structural configurations. It is also, possible in the near future to relate the structural system reliabilities to the element reliability.

However, it appears reasonable to account for the wind loading and enhance the value of yield stress. Also, it can be deduced that the two members are in two different configurations; therefore the combined effects of the random variables are different, altering the limit states and their distances from the origin. The observations also show that the reliability of a member is highly influenced by the structural configuration and that considering the effects of all the random variables is important.

CHAPTER VI

CONCLUSIONS AND FUTURE RESEARCH SUGGESTIONS

6.1   Conclusions

The study in this project covered the estimation of structural reliability under cumulative damage of single storey frame structures. The central idea was efficiently to impose the damage through a grouping operation, by exploiting the statistical correlations between modes of failures or by considering the amount of accumulated damage. Also it was sought to validate the load and resistance factors used in LRFD. The specific contribution of each finding is summarized and concluded as follows:

The validation process involved comparison of the reliability levels achieved by the actual structure to the target reliability levels set up according to the LRFD criteria. For numerical examples presented in this project, it is observed that the values of safety indices in the actual structure are higher than their target values. It is clearly seen from the fig 5-2, that if the framed structures are designed according to the LRFD format, it leads to a higher safety index than is desired. This helps in concluding

that the structures designed with this method are conservative. Furthermore, the statistical correlations between the sectional properties of different members are observed to have an insignificant effect on the effect on the system reliability. The location of a member in a structural configuration appears to be much more significant.

The LRFD method is based on the reliability analysis of simple, isolated members. Therefore, it does not consider the effect of the configuration on the stochastic response or reliability of a particular member; nor does it include the statistical correlations among all random variables of a structure. Using a finite element software like NESSUS offers the means to consider these factors and to estimate directly the reliability of the actual structural configuration. Therefore, this method can be used for a comprehensive validation of the LRFD approach, considering many different design situations. It is also possible to conduct sensitivity studies and compare the relative influence of various random parameters on the reliability.

Finally the occurrence of non-uniform safety indices among different structural transformations, suggests that an approach with the assignment of sizes of critical members in each design group should be followed, based on the reliability

of the overall structure or safety index of the overall
structure.

6.2 Suggestions for Future Research

Based on the results of the present study, the following
topics may be addressed in future research:

1. There is a need to use these results and similarly of
many different structural configurations to determine the
system affected element-reliability. In other words, the
design factors may be derived for the system affected element
reliability.

2. The proposed study gives a very approximate estimation
of system reliability under multiple time varying loads. The
incorporation of the geometry (dimensions) of the structure as
random variables would lead to a better result.

3. The component resistances are assumed to be time-
invariant. Practically the resistances are time varying due
to aging, material deterioration etc. Fu and Moses evaluated
the time dependent system reliability for a simple parallel
system by updating the probability distribution of the element
resistances at time t and using them to estimate system
reliability at this time. However, the inclusion of time
variant resistances in the estimation of system reliability

for a practical structure remains an important research topic.

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APPENDIX A

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C HIS PROGRAM CALCULATES THE REQUIRED MEAN RESISTANCE ; GIVEN THE
C BETA VALUE AND THE LOADING EFFECT WITH THE COEFFICIENTS OF VARIATIONS
C OF THE RESISTANCE AND LOAD EFFECTS.
                                          -NITISH BERI
                                          26TH SEPT 95
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

      IMPLICIT REAL*8 (A-H,O-Z)
      WRITE(*,*)' INPUT THE VALUE OF THE INDUCED MOMENT:(KIP-FT)'
      READ(*,*)R
      WRITE(*,*)' INPUT THE VALUE OF C.O.V OF THE MOMENT:'
      READ(*,*)Vr
      OPEN(UNIT=9,FILE='LNB.OUT',STATUS='UNKNOWN')
      WRITE(*,*)' INPUT THE VALUE OF PLASTIC MOMENT OF SECTION:'
      READ(*,*)Q
      WRITE(*,*)' INPUT THE VALUE OF C.O.V OF THE PLASTIC MOMENT:'
      READ(*,*)Vq
      OPEN(UNIT=9,FILE='LNB.OUT',STATUS='UNKNOWN')
      BETA1=(LOG(R/Q))/((Vr**2+Vq**2)**0.5)
      WRITE(*,*)'-------------------------------'
      WRITE(*,*)'  BETA = ', BETA1
      WRITE(*,*)'-------------------------------'
      WRITE(*,*)' IS THIS VALUE ACCEPTABLE ? (1=YES,0=NO)'
      READ(*,*)NUM1
      IF(NUM1.EQ.1.0)THEN
         GOTO 20
      ELSE
         GOTO 10
      ENDIF
      WRITE(9,*)'*******************************************************'
      WRITE(9,*)'  RESULTS FOR THE BETA AND RESISTANCES OF MEMBER     '
      WRITE(9,*)'                        -NITISH BERI  '
      WRITE(9,*)'*******************************************************'
      WRITE(9,*)' '
      WRITE(9,30)R
      WRITE(*,*)'*******************************************************'
      WRITE(*,30)R
      FORMAT('THE VALUE OF MEAN RESISTANCE MOMENT= ',F5.2, ' KIP-FT')
      WRITE(*,*)' '
      WRITE(9,*)'     '
      WRITE(9,40)Q
      WRITE(*,40)Q
      FORMAT(' THE VALUE OF THE IMPOSED LOAD= ',F5.2,' KIP-FT')
      WRITE(*,*)' '
      WRITE(9,*)' '
      WRITE(9,50)Vr
      WRITE(*,50)Vr
      FORMAT(' THE VALUE OF C.O.V OF RESISTANCE = ' ,F5.2)
      WRITE(*,*)' '
      WRITE(9,*)' '
      WRITE(*,60)Vq
```

```
      WRITE(9,60)Vq
60    FORMAT(' THE VALUE OF C.O.V OF LOAD MOMENT= ' ,F5.2)
      WRITE(*,*) '   '
      WRITE(9,*)'   '
      WRITE(*,70)BETA1
      WRITE(9,70)BETA1
0     FORMAT(' THE VALUE OF BETA USED = ' ,F5.2)
      WRITE(*,*)'*******************************************'
      WRITE(9,*)'*******************************************'
      STOP
      END
```

APPENDIX B

LOADING CALCULATIONS IN ACCORDANCE WITH THE UNIFORM BUILDING CODES:

According to the **Uniform Building Codes (UBC)**;

1. Minimum roof live load = 20 psf.

2. Minimum office live load = 50 psf.

Intermediating these values, we use;

Dead load on the frame = 20 psf.

Live load on the frame = 40 psf.

Choosing a bay width of 10 ft.

Uniform dead load on beams = 20*10 = 200#/ft = 0.2 K/ft

Uniform live load on beams = 40*10 = 400#/ft = 0.4 K/ft

Therefore,

Concentrated dead load at the center of beam = 3.6 K ≈ **4 K**

Concentrated live load at the center of beam = 7.2 K ≈ **7.5 K**

Basic Wind Speed = 70 mph. (Nashville)

$P = C_e \ C_q \ q_s \ I$

$I = 1.0$

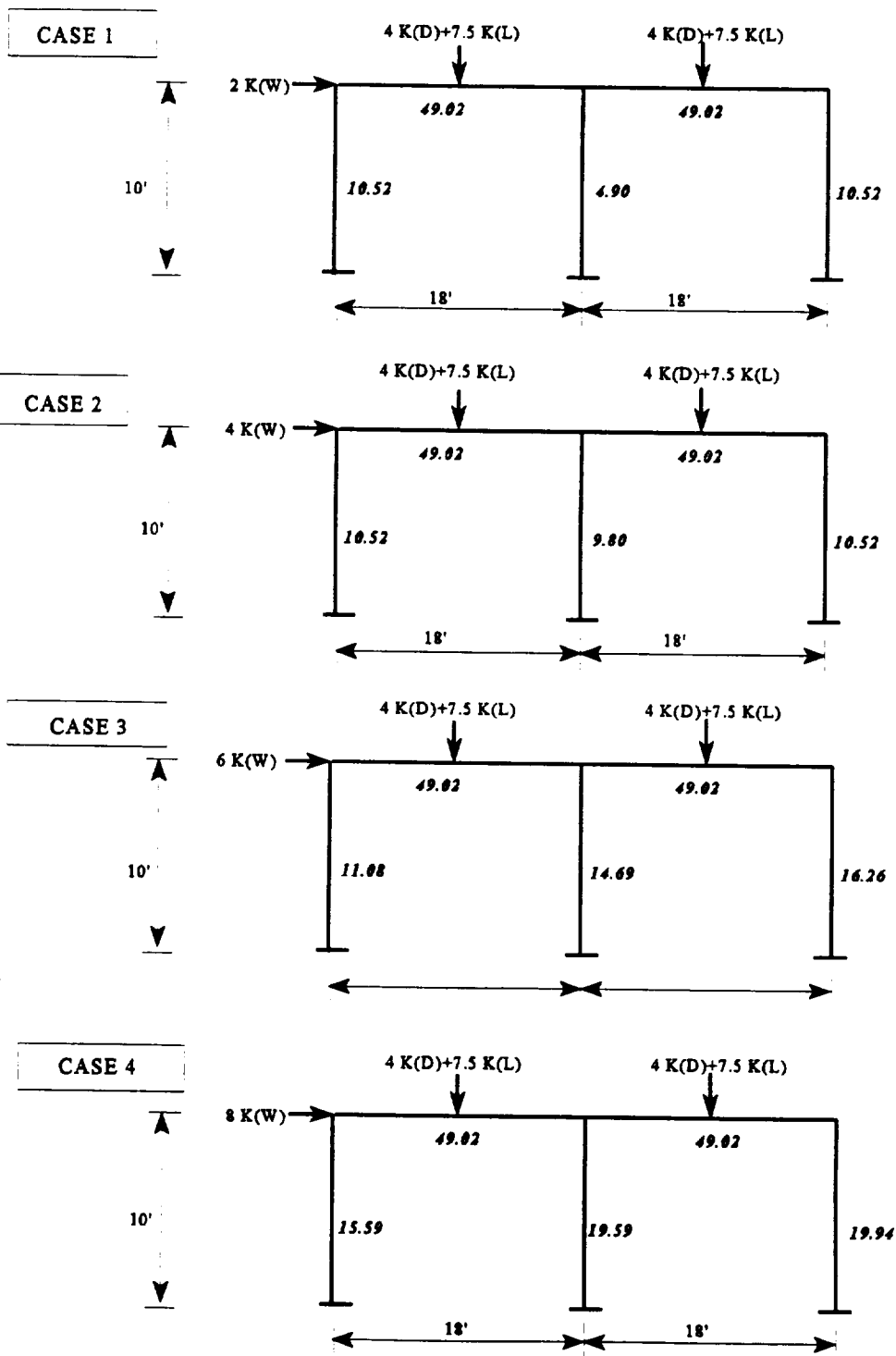$q_s = 12.6$ psf

C_e= 1.06 (Exposure C)

C_q= 1.2

Therefore,

P= 1.06*1.2*12.6*1.0

P= 16.03 psf.

Linear load along column edge = 16.1*1 = 161#/ft = 0.161 K/ft.

Horizontal wind load = 1.61 K ≈ **2 K**

APPENDIX C

CASE 1

4 K(D)+7.5 K(L)   4 K(D)+7.5 K(L)

2 K(W) →

49.02   49.02

10'

10.52   4.90   10.52

18'   18'

CASE 2

4 K(D)+7.5 K(L)   4 K(D)+7.5 K(L)

4 K(W) →

49.02   49.02

10'

10.52   9.80   10.52

18'   18'

CASE 3

4 K(D)+7.5 K(L)   4 K(D)+7.5 K(L)

6 K(W) →

49.02   49.02

10'

11.08   14.69   16.26

CASE 4

4 K(D)+7.5 K(L)   4 K(D)+7.5 K(L)

8 K(W) →

49.02   49.02

10'

15.59   19.59   19.94

18'   18'

100

CASE 5

6 K(D)+10 K(L)   6 K(D)+10 K(L)

2 K(W) →

68.28   *68.28*

14.64   *4.90*   *14.64*

10'

18'   18'

CASE 6

8 K(D)+12.5 K(L)   8 K(D)+125 K(L)

2 K(W) →

87.38   *87.38*

18.75   *4.90*   *18.75*

10'

18'   18'

CASE 7

10 K(D)+15 K(L)   10 K(D)+15 K(L)

2 K(W) →

106.56   *106.56*

22.87   *4.90*   *22.87*

10'

CASE 8

12.5 K(D)+17.5 K(L)   12.5 K(D)+17.5 K(L)

2 K(W) →

125.74   *125.74*

26.99   *4.90*   *26.99*

10'

18'   18'

CASE 9

4 K(D)+7.5 K(L)   4 K(D)+7.5 K(L)

2 K(W) →

49.66   49.66

12'   9.38   5.81   9.38

18'   18'

CASE 10

4 K(D)+7.5 K(L)   4 K(D)+7.5 K(L)

2 K(W) →

49.80   49.80

14'   8.45   6.70   9.57

18'   18'

CASE 11

4 K(D)+7.5 K(L)   4 K(D)+7.5 K(L)

2 K(W) →

50.06   50.06

16'   7.69   7.60   10.04

CASE 12

4 K(D)+7.5 K(L)   4 K(D)+7.5 K(L)

2 K(W) →

50.25   50.25

18'   7.06   8.48   10.58

18'   18'

CASE 13

4 K(D)−7.5 K(L)   4 K(D)+7.5 K(L)

2 K(W) →

54.17   *54.17*

10'

12.41   *4.94*   *12.41*

20'   20'

CASE 14

4 K(D)+7.5 K(L)   4 K(D)+7.5 K(L)

2 K(W) →

59.19   *59.19*

10'

14.42   *4.98*   *14.42*

22'   22'

CASE 15

4 K(D)+7.5 K(L)   4 K(D)−7.5 K(L)

2 K(W) →

64.10   *64.10*

10'

16.52   *5.02*   *16.52*

24'   24'

CASE 16

4 K(D)+7.5 K(L)   4 K(D)+7.5 K(L)

2 K(W) →

68.91   *68.91*

10'

18.72   *5.06*   *18.72*

26'   26'
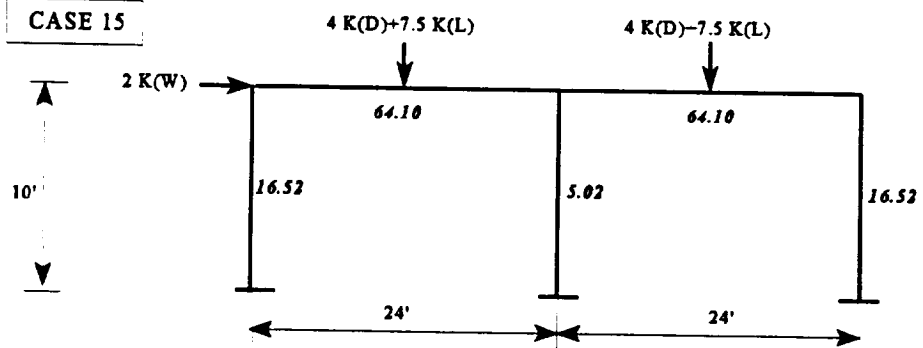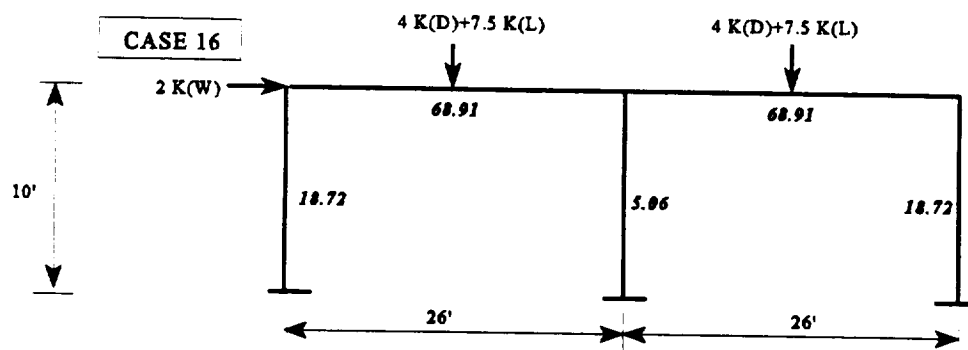
APPENDIX D

* **Design Algorithm for running NESSUS for Probabilistic Analysis**

(Single limit state problems)

1. Create a **data file** with a *.dat extension.

2. Copy the *.dat file to for000.dat. This can be done by typing **copy <*.dat> space for000.dat**.

3. To enter the failure function modify the subroutine **respon.for**

4. To edit the file respon.for, type **edt <respon.for>**. This will create an editor asterisk on the screen. Type 'c' at the '*' to get into full screen mode.

5. Make changes and exit the file by holding the ctrl key and pressing 'z'. This will again produce the editor's asterisk. Type **'save'** at the asterisk and close the file.

6. Once the subroutine is modified, it has to be compiled and linked to the library. To **compile** type **fortran <filename.for>**

7. Link the compiled file to the library by typing **link filename(omit extension), nes/lib**

8. The probabilistic analysis can be done by typing **run nessus** at the VAX prompt.

9. NESSUS will then ask for the filename. The filename should be typed **without** the .dat extension.

10. Once the run is completed, the output information will all be stored in for000.dat file. To preserve this information, change the name of the for000.dat to <input filename.out>, by typing **ren for000.dat space <input filename.out>**

11. To study the sensitivity analysis results, type **<input filename.mov>**. If the safety index is low or the probability of failure is high, identify the most sensitive design parameter from the sensitivity analysis.

12. Increase/decrease the coefficient of variance of the most important design parameter and do the probabilistic analysis again. This can be done by **repeating** steps 6 through 11.

* **Design Algorithm for Running NESSUS for Probabilistic Analysis (Multiple Limit state problem / System reliability)**

1. Create a **data file** with a *.dat extension.

2. Copy the *.dat file to for000.dat. This can be done by typing **copy <*.dat> space for000.dat.**

3. The probabilistic analysis can be done by typing **run**

nessus at the VAX prompt.

4. NESSUS will then ask for the filename. The filename should be typed **without** the .dat extension.

5. Once the run is completed, the output information will all be stored in for000.dat file. To preserve this information, change the name of the for000.dat to <input filename.out>, by typing **ren for000.dat space** <input **filename.out>**

6. Increase/decrease the coefficient of variance of the most the most sensitive design parameter and do the probabilistic analysis again. This can be done by **repeating** steps 1 through 5.

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                          for the work presented in this project.

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References: Available upon request.

STUDY OF DESIGN OF A GEAR TRAIN USING RELIABILITY DESIGN METHODS BASED ON OPTIMIZATION DESIGN METHODS

A Project

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of

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with a

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W.M.Z

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CHAPTER I

INTRODUCTION

1-1. Background

In modern engineering design, the need for designing high-reliability, optimal structure systems has been increasing recently due to the demand for greater quality, reliability, and lower cost or weight. An optimal structure system design must satisfy the performance of cost, volume, weight, or speed ratio objectives as well as the system or component reliability constraint. The latter is used to quantify the uncertainty existing in different failure models, loading conditions, material properties, and geometric parameters. To deal with these uncertainties, reliability technology provides tools for formal assessment and analysis. Meanwhile, optimization technology plays an important part to meet the optimal design objectives. Therefore, the combination of reliability and optimization technologies is a viable way to design high-reliability optimal structural systems.

The scope of this study is, however, mainly concentrated

1

on the design of a gear train using reliability-based optimization design method. Some failure models of gear, such as wear and thermal conditions, are not investigated in this study.

1-2. Objectives and Organization of the Report

2.1. The theory of the probabilistic design methodology in depth and an overview of the software, NESSUS (Numerical Evaluation of Stochastic Structure Under Stress), are described in Chapter II.

2.2. The theory of GRG (Generalized Reduced Gradient method for optimization) and its application, and development of GRG computer program for this project, are described in detail in Chapter III.

2.3. Discussion of the combination of reliability design method and optimization design method is presented in Chapter IV.

2.4. The application of reliability-based optimization design method for a gear train and the results, their interpretation, explanation and comparison are described in Chapter V.

2.5. The summary and conclusions of the present study and suggestions for future research are presented in Chapter VI.

CHAPTER II

PROBABILISTIC DESIGN METHODOLOGY

2-1 Introduction

In engineering designs, decisions are often required irrespective of the state of completeness and quality of information, and thus are made under conditions of uncertainty. In other words, the consequences of a given decision cannot be found out with complete confidence. Besides the fact that the information must often be inferred from similar circumstances or derived through modeling, many problems in engineering involve natural processes and phenomena that are inherently random. The states of such phenomena are naturally indeterminate and thus cannot be described definitely. For these reasons, decisions required during engineering design invariably must be made under conditions of uncertainty.

The effects of such uncertainty in design are important. To be sure, the quantification of such uncertainty and evaluation of its effects on the performance and design of an engineering system should include concepts and methods of probability. Furthermore, under conditions of uncertainty, the designs of engineering systems involve risks, and the

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formulation of related decisions requires them to be risk-free. The problems of uncertainty in the design can be overcome by applying the methods of probabilistic design. Thus, the role of probability is quite pervasive in engineering. It ranges from the description of information to the development of bases for design and decision-making [1].

PDM (Probabilistic Design Method) is concerned with the probability of non-failure performance of structures or machine elements. It is much more useful in situations in which design is characterized by complex geometry, possibility of catastrophic failure, or sensitive loads and material properties. Current studies on probabilistic structure analysis methods have resulted in a new class of tools that the engineer can use to obtain direct information on the uncertainty of structural performance. Reliability analysis evaluates the probability by a rational treatment of the uncertainties in various design parameters. It is becoming substantially evident that the PDM is beginning to attract more attention [2]. The PDM has been successfully applied to various loading conditions encountered during space flight [3]. Some reasons for the increasing acceptance of the PDM [2] are

1)   The deterministic method can provide some basic

information to complex design problems but provides no

information with regard to the reliability of the design.

2) Probabilistic computations are becoming simpler and less

expensive because of software being developed.

3) The PDM and the information it provides are becoming more

widely understood and better appreciated.

One of the most recent computer codes is NESSUS

(Numerical Evaluation of Stochastic Structure Under Stress).

This code was developed under NASA's probabilistic structure

analysis program. An overview of NESSUS and the description of

its development are given by Cruse et al.[4,5].

2-2. Application of PDM

Because probabilistic design method (PDM) is concerned

with the probability of failure-or preferably, reliability-it

is most useful when uncertainties in material properties and

loading conditions are considered. To apply probabilistic

design methodologies (PDM), all uncertainties are modeled as

random variables, with selected distribution types, means, and

standard deviations. The primitive (random) variables that

affect the structural behavior have to be identified. Every

design project demands some sequential stages of reflection

before one can arrive at the final design goal. This is also the case with PDM. The various design stages of PDM are as follows:

1) Defining the problem.

2) Generating design parameters.

3) Relating the defined problem to the design parameters.

4) Assembling data and applying probability concepts.

5) Using probabilistic Analysis.

6) Interpreting results.

The design stages of PDM are shown in Figure 2-1.
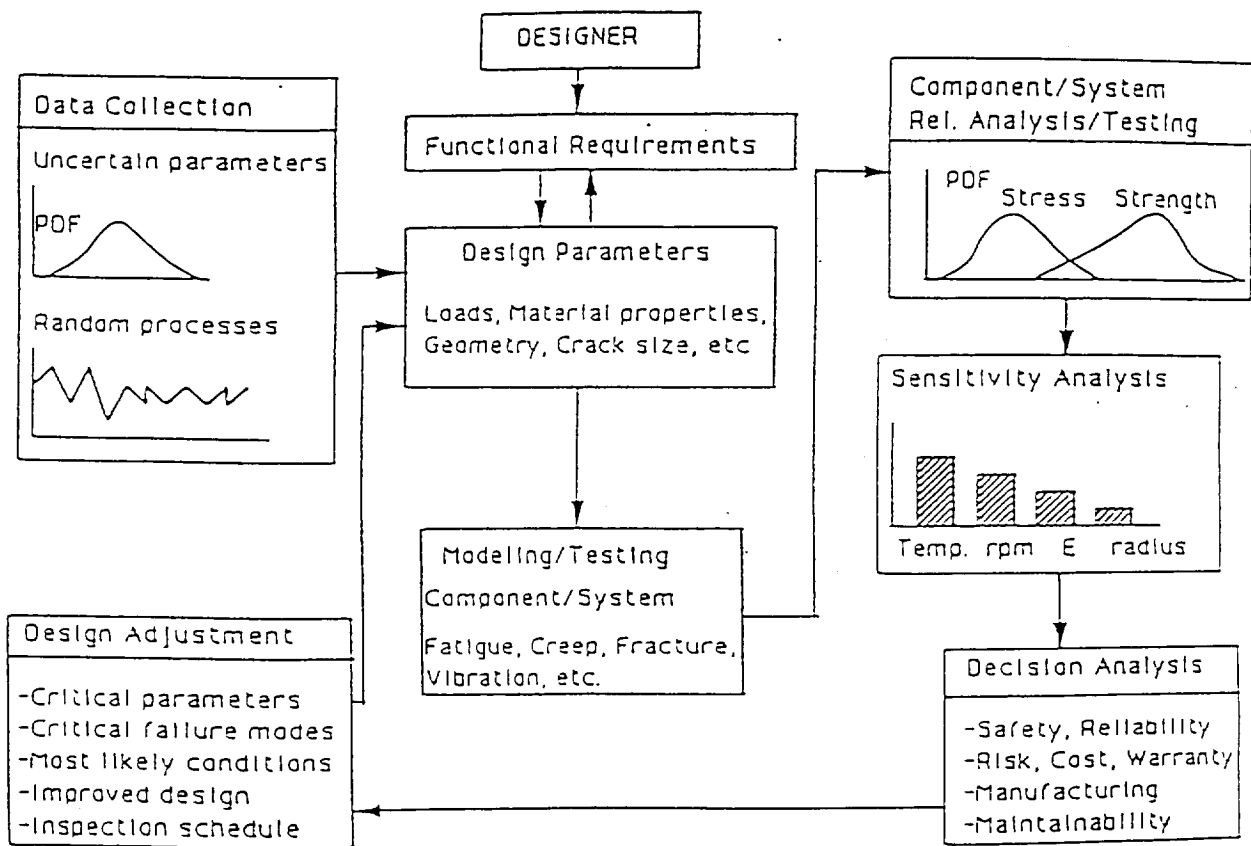
Figure 2-1: Design stages in PDM [6]

2-2.1. Problem definition

The first step which a designer takes in solving a design problem is to find out the main objective of the design. After finding out the objective, the next step is to define in a precise manner the functional requirements of the system or component to be designed. These functional requirements should be able to characterize completely the design objective by defining it in terms of specific needs. With a clear understanding of what one is searching for, the designer then goes to the next stage.

2-2.2. Generating design parameters

In order to solve the defined problem, acceptable design parameters must be generated that will meet the defined functional requirements. To generate the design parameters, one uses an appropriate design model. The various parameters (loads, material properties, geometry, etc.) are taken into consideration. The design parameters to be selected depend on the objective of the design [6].

2-2.3. Relating the defined problem to the design parameters

After defining the design parameters, the designer then relates the functional requirements in the functional domain to the design parameters in the physical domain, to be sure

that the objective is satisfied. If the relation is satisfactory, the designer goes to the next stage. If the relation is not satisfactory, it is redefined, so that the objective is satisfied.

2-2.4. Data assembling and application of probability concepts

This stage requires assembling the essential data that are available on the problem with regard to the design parameters. If some data are unavailable, then it becomes necessary to perform a computational simulation analysis to generate the missing details. Once the data have been assembled, the next stage is to analyze the assembled data. NESSUS is the computer tool used to perform the analysis. NESSUS has three modules, known as NESSUS/PRE, NESSUS/FEM, and NESSUS/FPI.

NESSUS/PRE is a preprocessor, which prepares the statistical data needed for the probabilistic design analysis. It allows the user to describe the uncertainties in the structural design parameters. The uncertainties in these parameters are specified by defining the mean value, the standard deviation, and the distribution type, together with an appropriate form of correlation. Correlated random variables are then decomposed into a set of uncorrelated vectors by a model analysis.

NESSUS/FEM is a general purpose finite element code, which is used to perform structural analysis and evaluation of sensitivity due to variation in different uncorrelated random variables. The response surface, defined in terms of random variables required for probabilistic analysis in NESSUS/FPI, is obtained from NESSUS/PRE. NESSUS/FEM incorporates an efficient perturbation algorithm to compute the sensitivity of random variables [6].

NESSUS/FPI is an advanced reliability module, which extracts the database generated by NESSUS/FEM to develop a response model in terms of random variables. In this module, the probabilistic structural response is calculated from the performance model. The probability of exceeding a given response value is estimated by a reliability method. Inside the NESSUS/FPI module is a sensitivity analysis program, which determines the most critical design parameters in the design. The input data for NESSUS/PRE requires fundamental knowledge of statistics or probability theorems. The expected details will include determining the mean, standard deviation, median, coefficient of variation, variances, etc., associated with each random variable. The designer also determines the probability distribution function that best describes each random variable. The different modules of NESSUS are shown in Figure 2-2.

```
┌─────────────────┐                    ┌──────────────┐
│  Nessus/Expert  │───────────────────→│   Random     │
└─────────────────┘                    │  Field data  │
        │                              └──────────────┘
        ↓                                      │
┌──────────────┐      ┌──────────────┐  ┌──────────────┐
│   Finite     │      │   Random     │──│  Nessus/PRE  │
│   Element    │      │   vectors    │  └──────────────┘
│   Model      │      └──────────────┘
└──────────────┘              │
        │                     ↓
        ↓             ┌──────────────┐      ┌──────────────┐
        └────────────→│  Nessus/FEM  │─────→│ Perturbation │
                      └──────────────┘      │  data base   │
                                            └──────────────┘
┌──────────────┐     ┌──────────────┐              │
│   User       │     │  Nessus/FPI  │←─────────────┘
│   queries    │     └──────────────┘
└──────────────┘             │
                             ↓
                     ┌──────────────┐
                     │  Analysis    │
                     │  results     │
                     └──────────────┘
```
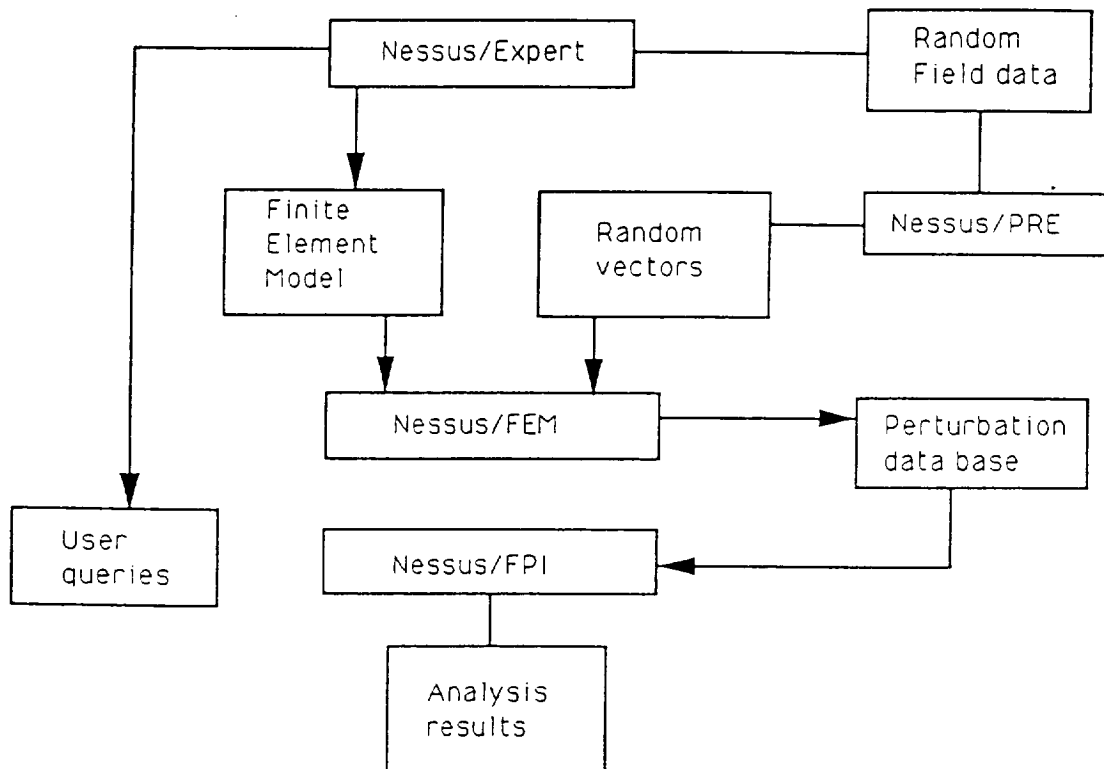
Figure 2-2: Modules of NESSUS [6]

2-2.5. Probabilistic Analysis

It is at this stage of the design that the designer defines a **limit state function**. The limit state function is a function that defines the boundary between the safe and failure regions. In the limit state function approach for structural reliability analysis, a limit state function $g(X)$ is first defined. The g-function is a function of a vector of basic random variables, $X = (X_1, X_2, X_3, . . . X_n)$ with $g(X)=0$ being the limit state surface that separates the design space into two regions, which are the failure $g(X)<0$ region and the safe $g(X)>0$ region. Geometrically, the limit state equation, $g(X)=0$, is an n-dimensional surface that may be called the "failure surface." One side of the failure surface is the safe state, $g(X)>0$, whereas the other side of the failure surface is the failure state, $g(X)<0$.

The probability of failure in the failure domain Ω is given by

$$P_f = \int_\Omega . . . \int f_x(X) \ dx \qquad (2\text{-}1)$$

where $f_x(X)$ is the joint probability density function of X and Ω is the failure region. The solution of this multiple integral is, in general, extremely complicated. Alternatively, a Monte Carlo solution provides a convenient but usually time-

consuming approximation. The limit state function method uses the Most Probable Point (MPP) search approach, shown in Figure 2-3. The Most Probable Point is the key approximation point for the FPI analysis; therefore, the identification of MPP is an important task. In general, the identification of the MPP can be formulated as a standard optimization problem and solved by proper optimization methods.

From the Figure 2-3, as the limit state surface, $g(X)=0$, moves closer to the origin, the safe region, $g(X)>0$, decreases accordingly. Therefore, the position of the failure surface relative to the origin of the reduced varieties should determine the safety or reliability of the system. The position of the failure surface may be represented by the minimum distance from the surface $g(X)=0$ to the origin. The point on the surface with minimum distance to the origin is the Most Probable Point (MPP). This is usually determined by fitting a local tangent to $g(X)$ and moving this tangent until MPP is estimated.

In the NESSUS code, MPP is defined in a transformed space called u-space where the u's are independent to simplify the probability computations. By transforming $g(X)$ to $g(u)$, the most probable point, u^*, on the limit state, $g(X)=0$, is the point that defines the minimum distance from the origin to the limit state surface. This point is most probable (in the

Figure 2-3: Illustration of Most Probable Point[6]

u-space) because it has maximum joint probability density on the limit state surface. The required minimum distance is determined as follows. The distance from a point $u^* = (u_1^*,\ u_2^*, \ldots,\ u_n^*)$ on the failure surface $g(u) = 0$ to the origin is

$$D \ = \ (\sum_{i=1}^{n} u_i^{*2})^{\frac{1}{2}} \tag{2-2}$$

where D is the minimum distance from the point on the limit state surface to the origin.

The FPI code assumes only one MPP. In general, however, the possibility exists that there may be multiple local and global Most Probable Points. A two MPP problem can occur; for example, if the g-function is quadratic, the search algorithm may result in oscillating (non-convergent) search.

Several approaches are available to search for the MPP. The search procedure depends on the forms and the number of the g-function(s). One efficient method in use is the Advanced Mean Value method (AMV). This method blends the conventional mean value method with the advanced structural reliability analysis method. This method provides efficient cumulative density function analysis and the reliability analysis. The step-wise AMV method can be summarized as follows [7]:

1. Obtain the g(X) function based on perturbations about

the mean values.

2. Compute the cumulative density function of the performance function at selected points using the fast probability integration method.

3. Select a number of cumulative density function values that cover a sufficiently wide probability range.

4. For each cumulative density function value, identify the most probable point.

The analytical process involved in the limit state approach can be illustrated by a basic structural reliability problem. In the problem, only one load effect, S, limited by one resistance, R, is considered.

If one considers a case when R and S are independent, the limit state equation can be expressed as

$$g = R - S \qquad (2\text{-}3)$$

and the probability of failure can be expressed as;

$$P_f = P(R - S \leq 0) = \int_{-\infty}^{\infty} \int_{-\infty}^{s} f_R(r) \, f_S(s) \, dr \, ds \qquad (2\text{-}4)$$

For any random variable the cumulative density function F(x) is given by

$$F_x(x) = P(X \le x) = \int f_x(Y)\,dy \qquad (2\text{-}5)$$

provided that x ≥ y. Therefore P_f is expressed as

$$P_f = P(R - S \le 0) = \int F_R(x)\,f_S(x)\,dx \qquad (2\text{-}6)$$

Assuming a special case of normal random variables, for some distributions of R and S, it is possible to integrate the equation (2-6) analytically and find out the probability of failure. If S and R have mean μ_R and μ_S and variance σ_R^2 and σ_S^2 respectively, the g-function has a mean μ_g and variance σ_g^2, given by

$$\mu_g = \mu_R - \mu_S \qquad (2\text{-}7)$$

$$\sigma_g^2 = \sigma_R^2 + \sigma_S^2 \qquad (2\text{-}8)$$

Therefore, the probability of failure is given as

$$P_f = P(R - S \le 0) = P(g \le 0) = \phi\left[\frac{0 - \mu_g}{\sigma_g}\right] \qquad (2\text{-}9)$$

Which reduces to:

$$\Phi \left[- \frac{(\mu_R - \mu_S)}{\sqrt{(\sigma_S^2 + \sigma_R^2)}} \right] = \Phi(-\beta) \qquad (2\text{-}10)$$

where β is defined as the safety index.

$$\beta = \frac{\mu_z}{\sigma_z} \qquad (2\text{-}11)$$

Thus, the probability of failure is given as

$$P_f = \phi(-\beta) \qquad (2\text{-}12)$$

which can be written as

$$P_f = 1 - \phi(\beta) \qquad (2\text{-}13)$$

Reliability is the probability that the structure will not violate a given performance criterion during a specified period. This can be mathematically expressed as

$$P_r = 1 - P_f \qquad (2\text{-}14)$$

where P_r is the reliability and P_f is the probability of

failure. Structural reliability analysis evaluates the probability of failure by rationally treating the various uncertainties.

2-2.6. Interpretation of Results

This is the last stage in the methodology. When the designer approaches this stage, one interprets the results obtained about the initial objective. If the results do not satisfy the functional requirements in the stage 1, the designer may adjust design parameters to achieve the set objective.

2-3. Probability Sensitivity Factors

In engineering performance analysis many sensitivity measures can be defined. Knowing the effect of each random variable in the analysis is important for the designer. The sensitivity information is quantified by sensitivity factors. Sensitivity factors suggest which random variables are crucial and require special attention.

The commonly used sensitivity factor in deterministic analysis is the performance sensitivity, $\partial Z/\partial X_i$, which measures the change in the performance due to the change in a design

parameter. This concept can be extended to the probabilistic analysis in which a more direct sensitivity measure is the reliability sensitivity that measures the change in the probability/reliability relative to the distribution parameters such as the mean and the standard deviation. Although not automated in the code, this analysis can be performed by varying the parameters.

Another, perhaps more important, kind of probability or reliability sensitivity analysis is the determination of the relative importance of the random variables. This analysis can be done, for example, by repeated probabilistic analysis in which one random variable at a time is treated as a deterministic variable. The results of the analyses, for example, are a number of cumulative density function curves or reliabilities. Based on the results, the relative importance of the random variables can be analyzed. The standard FPI output includes a first-order sensitivity factor that provides approximate relative importance of the random variables.

CHAPTER III

OPTIMIZATION DESIGN METHODOLOGY

3-1. Introduction

Optimization is the method of obtaining the best result under given circumstances. In design, construction, and maintenance of any engineering system, engineers have to take many technological and managerial decisions at several stages. The ultimate goal of all such decisions is either to minimize an effort required or to maximize a desired benefit. Engineering design is a multiphase process requiring constant decision making by the designer. Based on his decision, the engineer is able to define variables, a design objective, and a set of constraints that must be met in order that the design is a workable solution. By developing corresponding equations, the design problem can be formulated into a standard form acceptable to mathematical programming techniques. This standard form is defined below.

Minimize $\qquad$ f(X)

$$X = (x_1, x_2, x_3, \ldots, x_N)^T, \quad X \in R^N \qquad (3-1)$$

subject to

$$\phi_k(X) \geq 0 \qquad k = 1,2,3,...,K \qquad\qquad (3\text{-}2)$$

$$\psi_l(X) = 0 \qquad l = 1, 2, 3,..., L \qquad\qquad (3\text{-}3)$$

where

      X is a column vector of design variables

      N is total number of design variables

      $f(X)$ is objective function

      $\phi_k(X)$ is K inequality constraint functions

      $\psi_l(X)$ is L equality constraint functions

A more general occurrence in engineering design arises when expressions (3-1 to 3-3) are nonlinear. This is known as the nonlinear programming (NLP) problem. No general method has been developed to solve nonlinear problems in the sense that the simplex algorithm exists to solve the linear problem. Although many strategies have been suggested, comparative studies [8,9] have shown that no method has been successfully applied to all problems. In this project the Generalized Reduced Gradient (GRG) method will be described. The GRG

method avoids many of the problems associated with penalty function and LP-like methods [10], producing one of the most powerful methods currently known for handling the constrained nonlinear programming problem. The principle behind this method is quite simple, but its application is rather complex [11].

3-2. Theory of Generalized Reduced Gradient Method

The Reduced Gradient method was originally given by Wolfe for a nonlinear objective function with linear constraints [12,13]. A generalization of Wolfe's method to accommodate nonlinearities in both the objective function and constrains was first accomplished by Abadie [14]. Concurrently to both Wolfe and Abadie, Wilde and Beightler developed their differential algorithm based on the constrained derivative [15]. The constrained derivative and the reduced gradient employ much the same theoretical basis, but for purposes of this discussion, the method shall be known as the Reduced Gradient method. The case of nonlinear constraints was pioneered by Abadie [14], who called it the "generalized reduced gradients (GRG)." Later variants were developed by Lasdon and Waren [16], Gabriele and Ragsdell [10]. Both Gabriele [17] and Lasdon and Waren have implemented versions for large sparse systems. The general constrained nonlinear

programming can be stated in the following form:

Minimize $\qquad$ $f(\mathbf{X})$

$$\mathbf{X} = (x_1, x_2, x_3, ..., x_N,)^T, \quad \mathbf{x} \in \mathbf{R}^N \qquad (3 - 4)$$

subject to

$$\psi_m(\mathbf{X}) = 0 \qquad m = 1, 2, ..., M \qquad (3 - 5)$$

$$A \leq \mathbf{X} \leq B \qquad (3 - 6)$$

The N × 1 vectors A and B represent upper and lower bounds on the design vector $\mathbf{X}$. These upper and lower bounds can be assumed to be the finitive or infinitive bounds. The inequality constraints have been included as equality constraints by using the following transformation:

$$\psi_k(\mathbf{X}) = \phi(\mathbf{X}) - S_k = 0$$

$$0 \leq S_k \leq \infty \qquad k = 1, 2, ..., K \qquad (3-7)$$

If $\phi(\mathbf{X}) \leq 0$, the equation (3-7) will be changed to

$$\psi_k(\mathbf{X}) = \phi(\mathbf{X}) + S_k = 0$$

$$-\infty \leq S_k \leq 0 \qquad k = 1,2,\ldots,K \qquad (3-8)$$

The variables S_k are slack variables that are included in the original set of design variables. Therefore, the parameter N represents the total number of design variables plus the number of slack variables used for the transformation of (3-7) or (3-8). The parameter M represents the total number of constraints:

$$M = L + K \qquad (3-9)$$

Where

    L is number of equality constraints

    K is number of inequality constraints

It should be stressed that the constraints of (3-5) are nontrivial constraints; that is, they are functional constraints. Variable bounds are defined in (3-6) and will require separate handling.

Linearization of equation (3-5) will result in M equality constraints with N independent variables. If the constraints were linear, all we have to do is to use elimination process

to reduce the number of independent variables to K using the equality constraints and then substituting the independent variables into the objective function f(**x**). Unfortunately, the problem is nonlinear so direct substitution is very difficult. Consider the following strategy whose fundamentals can be found in the simplex method of linear programming. Divide the design vector of equation (3-9) into two classes that shall be known as the decision and state variables.

$$\mathbf{X} = [\ \mathbf{Z} \ , \ \mathbf{Y} \ ]^{\mathrm{T}} \tag{3-10}$$

$$\mathbf{Z} = [\ z_1, z_2, \dots z_Q \ ]^{\mathrm{T}} \tag{3-11}$$

$$\mathbf{Y} = [\ y_1, y_2, \dots y_M \ ]^{\mathrm{T}} \tag{3-12}$$

where

    Z : decision variables; **y** : state variables.

    Q : number of decision variables, $Q = N - M$.

The decision variables are completely independent, and the state variables are slaves to the decision variables used to satisfy the constraints $\psi(\ \mathbf{X}\ )$.

The following notation will be useful in the discussion to follow:

$$g(Y) = [\ \frac{\partial f(X)}{\partial y_1} \ , \ \frac{\partial f(X)}{\partial y_2} \ , \dots , \ \frac{\partial f(X)}{\partial y_M} \ ]^{T} \tag{3-13}$$

$$g(Z) = [\ \frac{\partial f(X)}{\partial z_1} \ , \ \frac{\partial f(X)}{\partial z_2} \ , \ \ldots \ , \ \frac{\partial f(X)}{\partial z_Q} \ ]^T \qquad (3\text{-}14)$$

$$\frac{\partial \psi}{\partial Z} = \begin{bmatrix} \dfrac{\partial \psi_1}{\partial z_1} & \dfrac{\partial \psi_1}{\partial z_2} & \cdot \ \cdot & \dfrac{\partial \psi_1}{\partial z_Q} \\ \cdot & \cdot & \cdot \ \cdot & \cdot \\ \cdot & \cdot & \cdot \ \cdot & \cdot \\ \dfrac{\partial \psi_M}{\partial z_1} & \dfrac{\partial \psi_M}{\partial z_2} & \cdot \ \cdot & \dfrac{\partial \psi_M}{\partial z_Q} \end{bmatrix}_{(M \times Q)} \qquad (3\text{-}15)$$

$$\frac{\partial \psi}{\partial Y} = \begin{bmatrix} \dfrac{\partial \psi_1}{\partial y_1} & \dfrac{\partial \psi_1}{\partial y_2} & \cdot \ \cdot & \dfrac{\partial \psi_1}{\partial y_M} \\ \cdot & \cdot & \cdot \ \cdot & \cdot \\ \cdot & \cdot & \cdot \ \cdot & \cdot \\ \dfrac{\partial \psi_M}{\partial y_1} & \dfrac{\partial \psi_M}{\partial y_2} & \cdot \ \cdot & \dfrac{\partial \psi_M}{\partial y_M} \end{bmatrix}_{(M \times M)} \qquad (3\text{-}16)$$

Let us examine the first variation of f(X) and ψ(X),

$$df = g(Z)^T \ dZ + g(Y)^T \ dY \qquad (3\text{-}17)$$

$$d\psi = \frac{\partial \psi}{\partial Z} dZ + \frac{\partial \psi}{\partial Y} dY \equiv 0 \qquad (3\text{-}18)$$

where

dZ = Q×1 vector of differential displacements of Z

dY = M×1 vector of differential displacements of Y

Solving (3-17) and (3-18) and rearranging will yield the following linear approximation to the reduced gradient:

$$dY = -\frac{\partial \psi}{\partial Y}^{-1} \frac{\partial \psi}{\partial Z} dZ \qquad (3\text{-}19)$$

Substituting (3-19) into (3-17)

$$g_r(X)^T = g(Z)^T - g(Y)^T \frac{\partial \psi}{\partial Y}^{-1} \frac{\partial \psi}{\partial Z} \qquad (3\text{-}20)$$

The reduced gradient defines the rate of change of the objective function with respect to the decision variables with the state variables adjusted to maintain feasibility. Expression (3-19) gives the changes necessary in the states for a given change in the decisions for linear constraints. Geometrically the reduced gradient can be described as a

projection of the original N - dimensional gradient onto the (N - M) - dimensional feasible region described by the decision variables.

A necessary condition for the existence of a minimum of an unconstrained nonlinear function is that the elements of the gradient vanish. Similarly, a minimum of the constrained nonlinear function occurs when the appropriate elements of the reduced gradient vanish. This conclusion can be verified by a comparison with the Kuhn - Tucker [18] conditions for the existence of a constrained relative minimum.

By first transforming the variable bounds into inequality constraints,

$$\phi_i(\mathbf{X}) = x_i - a_i \geq 0 \qquad (3\text{-}21)$$

$$\phi_{i+N}(\mathbf{X}) = b_i - x_i \geq 0 \qquad (3\text{-}22)$$

where i = 1,2,3,...,N

we can form the following lagrangian function:

$$L(X,U,W) = f(X) + \sum_{m=1}^{M} W_m \, \psi_m(X) - \sum_{j=1}^{2N} U_j \, \phi_j(X) \qquad (3\text{-}23)$$

The following Kuhn-Tucker necessary conditions hold for a

point to be a relative minimum X^*,

$$g(X^*)^T + \sum_{m=1}^{M} W_m^* \frac{\partial \psi_m}{\partial X} - \sum_{j=1}^{J} U_j^* \frac{\partial \phi_j}{\partial (X)} = 0 \qquad (3\text{-}24)$$

and

$$\psi_m(X^*) = 0 \qquad m = 1,2,\ldots,M \qquad (3\text{-}25)$$

$$\phi_j(X^*) \geq 0 \qquad j = 1,2,\ldots,J = 2N \qquad (3\text{-}26)$$

$$U_j^* \phi_j(X^*) = 0 \qquad j = 1,2,\ldots,J = 2N \qquad (3\text{-}27)$$

$$U_j^* \geq 0 \qquad j = 1,2,\ldots,J = 2N \qquad (3\text{-}28)$$

$$W_m^* \neq 0 \qquad m = 1,2,\ldots,M \qquad (3\text{-}29)$$

Introducing decision and state variables into (3-24) and decomposing we can obtain the following form:

$$g(Z^*)^T + W^{*T} \frac{\partial \psi}{\partial Z^*} - U^{*T} \frac{\partial \phi}{\partial Z^*} = 0 \qquad (3\text{-}30)$$

$$g(Y^*)^T + W^{*T} \frac{\partial \psi}{\partial Y^*} - U^{*T} \frac{\partial \phi}{\partial Y^*} = 0 \qquad (3\text{-}31)$$

For the reasons that a state variable is not allowed to be equal or sufficiently close to either of its bounds. Form expression (3-27) the elements of U^* corresponding to the state variable bounds must be zero. Also, those elements of

$\partial \phi$ /∂Y corresponding to the decision variable bounds will be zero eliminating the last term of (3-31). Solving (3-31) for W^* and substituting into (3-30) will produce the following expression:

$$g\ (\ Z^*\ )^T - g\ (\ Y^*\ )^T\ \frac{\partial \psi^{-1}}{\partial Y^*}\ \frac{\partial \psi}{\partial Z^*} - U^{*T}\ \frac{\partial \phi}{\partial Z^*} \equiv 0 \qquad (3\text{-}32)$$

Rearranging (3-32), we obtain

$$U^{*T}\ \frac{\partial \phi}{\partial Z^*} = g\ (\ Z^*\ )^T - g\ (\ Y^*\ )^T\ \frac{\partial \psi^{-1}}{\partial Y^*}\ \frac{\partial \psi}{\partial Z^*} \qquad (3\text{-}33)$$

It can be recognized that the right-hand side of (3-33) is $g_r(X)$. By examining the possible values of the left-hand side of (3-33), a candidate point X will be X^* if

$$g_r(X)_i > 0 \text{ if } z_i = a_i$$
$$g_r(X)_i < 0 \text{ if } z_i = b_i$$
$$g_r(X)_i = 0 \text{ if } a_i \leq z_i \leq b_i$$

where $\qquad\qquad i = 1,2,3,\ldots,Q \qquad\qquad (3\text{-}34)$

3-3. Generation of Feasible Starting Points

In the application of GRG method, generation of feasible starting points is an important step. As the final element of problem presolution analysis, the formulation should be tested for feasibility. While the preceding stages of problem preparation and analysis may have resulted in a numerically stable, bounded, and nonredundant formulation, it is always possible that, as a result of poor data or calculation errors, the problem constraints simply exclude all possible solutions. Thus, whether or not the optimization algorithm selected for use requires a feasible starting point, it is good practice to devote some effort to generating a feasible starting point. Obviously, if no feasible starting point can be obtained, there is little point in proceeding with optimization. Instead, the model must be inspected again and validated in a piecemeal fashion until the sources of error are identified. If a feasible point can be generated, and if the variable values at the generated point appear reasonable, then one can proceed with problem solution with a fair degree of confidence that the optimization runs will be productive.

A very common way of generating feasible starting points is direct minimization of the constraint infeasibilities. The method of minimization of unconstrained penalty-type functions has been proposed. This method is often preferable,

especially, for higher dimensionality and tightly constrained problems [19]. The procedure consists of solving an unconstrained minimization problem whose objective function is an exterior penalty function. The starting point is thus obtained as the solution of the problem.

Minimize:

$$f(X) = \sum_{i=1}^{k} (\psi_i (X))^2 + \sum_{j=1}^{N} (\min (0, \phi_j (X)))^2 \qquad (3\text{-}35)$$

where

$\psi_i (X)$ = quality constrained functions.

k = number of quality constrained functions.

$\phi_j (X)$ = inequality constraints of the variable bounds.

N = number of unknown variables in function.

Clearly a feasible point is one that will result in an objective function value of zero. Hence, the unconstrained minimization will be terminated when f(X) becomes sufficiently small. Generally, the minimization can be simplified if the problem is posed in equality-constraint-free form.

3-4. Perform the Line Search to Locate Local Minimum

When a search direction D(Y) is determined for the state

variables, the vector D(Z) has defined a line in the reduced N - M dimensional space along which exists a local minimum of the objective function. It is the task of this section to locate the minimum so that it might be used as a starting point for the next iteration. The performance of this task is a common occurrence in many unconstrained searching techniques, and in the case of the reduced gradient represents the bulk of the computational effort.

The normal course of events in locating a minimum along a line consists of two phases. The first phase involves locating an initial bracket within which the minimum is known to be contained. This is commonly referred to as the bounding phase. The second phase would consist of some efficient scheme of narrowing the initial bracket unit the minimum is known to be within some tolerance. Both these phases are outlined in more detail in [20,21,22].

The Reduced Gradient Method uses this same two - phase procedure with modifications to accommodate the use of state and decision variables. From a starting point $(Z,Y)^k$ we move to a new point $(Z,Y)^{k+1}$ according to the step prescription

$$z_i^{k+1} = b_i \quad \text{if } z_i^k + \alpha \, D(z_i) \geq b_i$$

$$z_i^{k+1} = a_i \quad \text{if } z_i^k + \alpha \, D(z_i) \leq a_i$$

$$z_i^{k+1} = z_i^k + \alpha \, D(z_i) \quad \text{otherwise}$$

where $\quad$ i = 1,2,3,...,Q $\hspace{5cm}$ (3-36)

and

$$Y_m^{k+1} = Y_m^k + \alpha \, D(Y_m) \quad m = 1,2,...,M \hspace{1.5cm} (3.37)$$

where α = step length parameter.

Because of nonlinearities arising in the constraint functions, the point $(Z,Y)^{k+1}$ is likely to be infeasible. Holding the decision variables Z^{k+1} constant, the state variables Y^{k+1} are adjusted to obtain a feasible point, $(Z,Y)^{k+1}$. This situation is shown in Figure 3-1 for the case M = 1, Q = 1. This step is equivalent to the solution of M nonlinear equations $(\psi(X) = 0)$ in M unknowns (Y). A number of numerical techniques are available in the literature to perform this task. Newton's method [23] has proven to be an efficient technique as well as convenient since the necessary partial derivatives have already been calculated. At the completion of the adjustment procedure, a new point $(Z,Y)^{k+1}$ has been determined, and the following possible results must be considered:

(a). If all elements of Y^{k+1} are within their specified bounds, then f(X) is evaluated at $(Z,Y)^{k+1}$, and the procedure for determining the minimum continues in the normal manner.
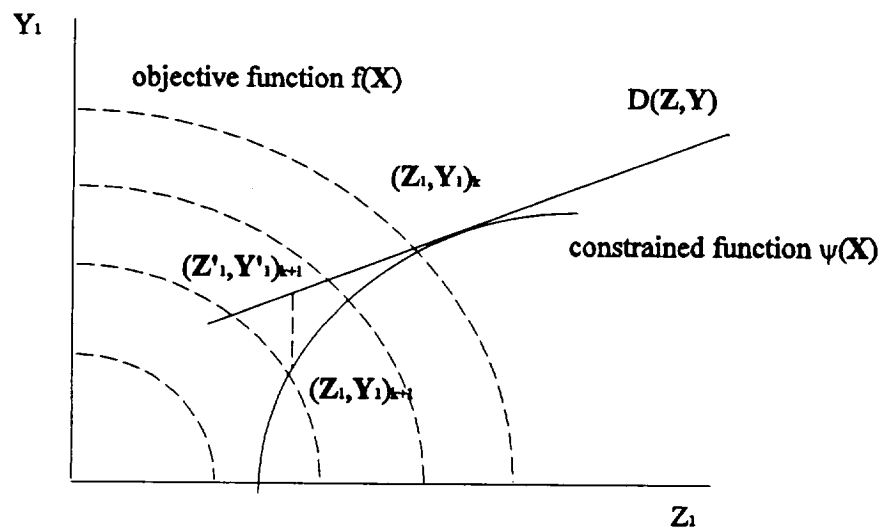
Figure 3-1. Adjustment of state
variable to obtain a feasible
point during the linear search

(b). If any element of Y^{k+1} is not within its specified bounds, then $(Z,Y)^{k+1}$ is infeasible. Successive linear interpolation is performed between the last feasible point $(Z,Y)^k$ and the point $(Z,Y)^{k+1}$ to determine the step length at which the nearest bound becomes active. Hence this step should conclude with a single state variable equal to one of its bounds and all other state variables within their specified bounds. Figure 3-2 shown $(z_1,y_1)^2$ being out of bounds $y_1 < 0$). Using successive false position, the bound point $(z_1,y_1)^3$ can be located. Supplementary tests are then performed to determine whether the local minimum lies at the bound or at some point before it. If the minimum lies at the bound, then the line search is terminated. If it lies before the bound, then the minimum has been bracketed, and refinement can be started to locate the minimum.

(c). If the procedure fails to converge in a reasonable amount of time, the step length is reduced $(\alpha = \alpha/2)$, and a new trial point is generated.
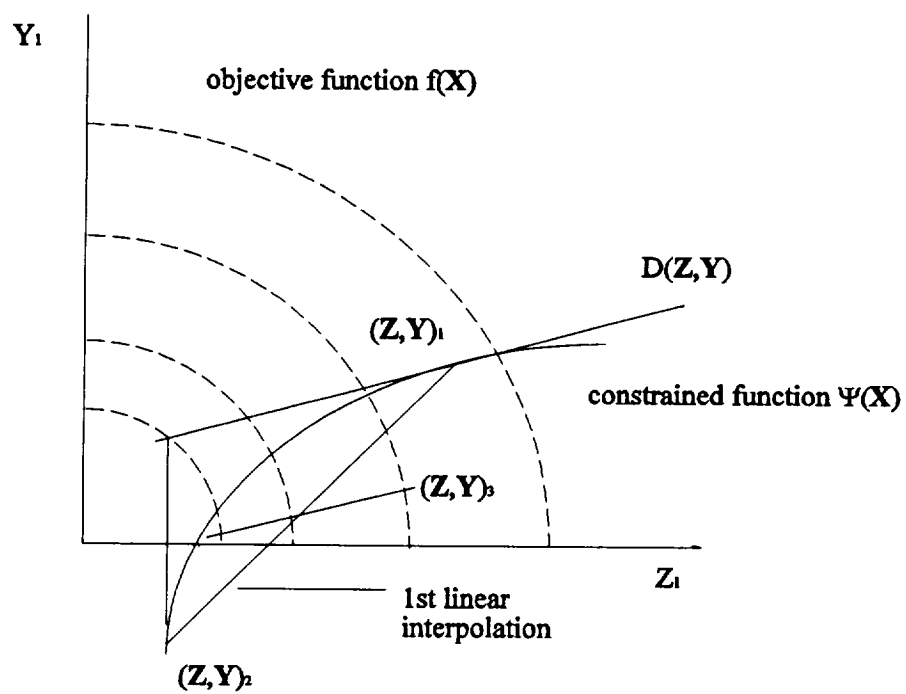
Figure 3-2. Adjustment of state
variable to locate a feasible
point.

3.5. Algorithm of Generalized Reduced Gradient Method.

According to the principle of GRG method, its algorithm is presented as the following:

Step 1. Obtain the feasible initial points.

Given a specified initial value of the search parameter $\alpha = \alpha^0$, termination parameter ϵ .

Step 2. Choose a partition of X into state Y and decision Z variables such that $\partial\Psi/\partial Y$ has nonzero determinant.

Step 3. Calculation of the reduced gradient D : the direction of move for the independent variable z , by the following substeps:

Step 3.1. Compute the reduced gradient D(z), given by:

$$g_j(X)^T = g(Z)^T - g(Y)^T \left(\frac{\partial\Psi}{\partial Y}\right)^{-1} \left(\frac{\partial\Psi}{\partial Z}\right)$$

$$D(z_j) = 0 \quad \text{if} \quad z_j = a_j \quad \text{and} \quad g_j > 0$$
$$D(z_j) = 0 \quad \text{if} \quad z_j = b_j \quad \text{and} \quad g_j < 0$$
$$D(z_j) = g_j \quad \text{otherwise}$$

if $\sum D(z) \leq$ termination parameter ϵ, then a constrained relative minimum has been obtained. Otherwise, the algorithm proceeds to the next step.

Step 3.2. Compute **D(y)**, the modified reduced gradient, i.e. the (opposite) direction of move for the independent variable **y**. This direction may simply be **D(y)**:

$$D(Y) = - \frac{\partial \Psi}{\partial Y}^{-1} \frac{\partial \Psi}{\partial Z} D(Z)$$

Step 4. Compute a first value of the positive number α . and Compute $z^0 + \alpha$ **D**, and project it onto the parallelotope $a_j \leq z_j \leq b_j$ to obtain z^1 . set :

$$z_j^1 = a_j \quad \text{if} \quad z_j^0 + \alpha D_j < a_j$$
$$z_j^1 = b_j \quad \text{if} \quad z_j^0 + \alpha D_j > b_j$$
$$z_j^1 = z_j^0 + \alpha D_j \quad \text{otherwise}$$

Step 5. Compute a feasible Z^1 corresponding to α , i.e. try to solve, with respect to **y**, the system of M equations in M unknowns:

$$\psi (z^1 , y) = 0$$

This is usually done by some iterative method (Newton'method).

Step 6. If no speedy convergence is observed, then decrease α for instance, $\alpha_1 = 1/2 \alpha$) and go to step 4, with the same **D**. Otherwise, let y^1 be the solution obtained

for $\psi(z^1, y) = 0$, and Z^1 the corresponding point in the whole n - dimensional space,

Step 7. If $f(Z^0) < f(Z^1)$, then decrease α, as above, and go to step 4, with the same **D**. If α = specific criterion such as 10^{-12}, then go to step 2. Otherwise, at the end of step 7, we have some feasible Z^1, which satisfies :

$$f(Z^1) < f(Z^0)$$

Step 8. We may now, either set $Z^0 = Z^1$ and begin a new iteration, or try to improve the last value obtained for α. In doing this, we return to step 4 for any new value tried for α, with the same **D**, and eventually terminate step 7 with some Z^1 satisfying $f(Z^1) < f(Z^1)$, and then begin a new iteration with $Z^0 = Z^1$, go to step 3.
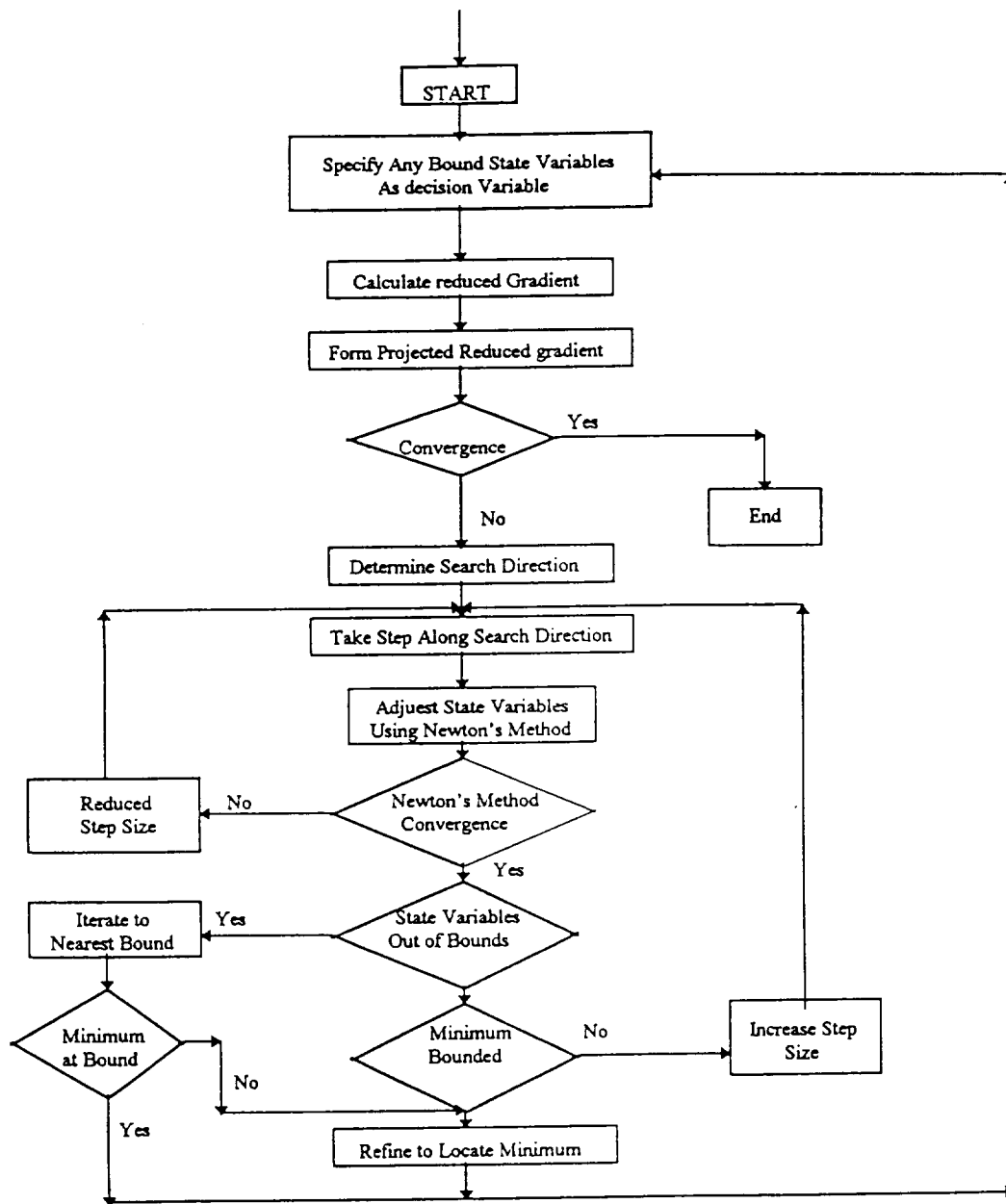
Figure 3-3 Flow chart for generalized reduced gradient (GRG) optimization method

CHAPTER IV

RELIABILITY DESIGN METHOD BASED ON

OPTIMIZATION DESIGN METHOD

4-1.Introduction

The ultimate goal in engineering design is to produce an optimal structure system that satisfies the performance/cost/ weight/volume/speed ratio objectives as well as the system or component reliability constraint, which is used to account for uncertainty existing in different failure models, loading conditions, material properties, and geometric parameters. To deal with these uncertainties, reliability technology provides tools for formal assessment and analysis of such uncertainties. However, in order to reach the optimal design objective, an appropriate optimizer must be used. The reliability design method based on optimization design method (also called as Probabilistic Design Optimization [PDO]) has been researched by Frangopol[24]; Sorensen and Thoft-Christensen [25]; Nicolaidis and Burdisso [26]; Maglaras and Nikolaidis [27]; Torng and Yang [28]; and Onwubiko et al.[29].

The objective function for optimal design, OBJ(**X**), is

subject to the following constraints:

$$P(g_i(\mathbf{X}) \geq 0) \geq (1 - \rho_i), \quad i = 1, \ldots, L. \qquad (4-1)$$

Where $\mathbf{X}$ is the vector of n random variables, $P(.)$ denotes the probability of the event $(.)$, g_i represents the i-th limit state function, $(1 - \rho_i)$ is the reliability goal for the i-th constraint or failure mode, and L represents the total number of constraints.

In general, there are two major difficulties for the design problem: (1) how to solve complex problems that require a computation intensive program and (2) how to reduce the total computational effort within the design optimization process. To overcome these difficulties, the proposed method uses an advanced mean value method (AMV) [30, 31] which has been illustrated to be efficient for solving reliability for complex problems. To improve the efficiency of computation, the proposed method uses an approximate function to represent the original complex component reliability problem [32]. In other words, there will be only one reliability calculation in each design iteration.

4-2. Optimal Structural Design Definition :

An optimal structural system design must be insensitive

to uncertainties incurred from material properties, environmental conditions, manufacturing variations,etc. A smaller system variation must be achieved in order to reduce the possible failure [33,34]. In other words, this optimal structural system design must have higher reliability or lower probability of failure. An optimal structural system design is defined as a high reliability system which not only satisfies the performance weight / volume / cost objective but also the component / system reliability constraints.

To achieve an optimal structural system design, the first important thing is to have a well-defined design problem. With consideration of design random variables, a more optimal structural system can be achieved; however, the optimal design problem setup must be redefined. In general, this new design optimization problem will have an objective function to be minimized or maximized as follows:

$$\text{Objective :} \qquad F(\mathbf{X},\mathbf{Y}), \qquad\qquad (4\text{-}2)$$

Subject to the reliability constraints:

$$P(g_i(\mathbf{X},\mathbf{Y}) \geq 0) \geq (1-\rho_i), \quad i=1,\ldots,L. \qquad (4\text{-}3)$$

Where $\mathbf{Y}$ is the vector of m design random variables, $\mathbf{X}$ is the

vector of n random variables, P(.) denotes the probability of the event(.), g_i represents the i - th limit state function, $(1 - \rho_i)$ is the reliability goal for i-th constraint or failure mode, and L represents the total number of constraints.

4-3. Reliability Constraint Function Definition

With all the random variables or design random variables defined, different failure mechanisms - e.g., yield failure, fracture failure, and so on - need to be established. Reliability constraint functions or limit state functions are used to represent these failure mechanisms. These functions can be constructed through the response function, Z,

$$Z = Z(\ X,Y\ ) \tag{4-4}$$

where X represents the random variables and Y represents the design random variables. This Z function can be a simple close form function or a complicated model which requires the use of computer intensive program to model [30].

To calculate the reliability or probability of failure, a critical failure event must be defined. This failure event is defined when Z function value is less than or greater than a critical response value z_o. In other words, the reliability

constraint function or limit state function, g, becomes:

$$g(\ X,Y\ ) = Z(\ X,Y\ ) - z_o \qquad (4\text{-}5)$$

The limit state $g(X) = 0$ separates the variable space into "failure" and " safe" regions. When the equal chance constraint function becomes unequal, i.e., $g < 0$ or $Z \leq z_o$ the reliability or probability of failure, p_f can be calculated as :

$$P_f = \text{Prob}(g(X,Y) \leq 0) = \text{Prob}(Z(X,Y) - z_o \leq 0) \qquad (4\text{-}6)$$

For each simple close formed g function, the reliability computation is straightforward. To calculate an implicitly defined g-function, however, the total computation becomes time consuming so that the selected probabilistic method must be efficient and reasonably accurate.

4.4. Reliability Constraint Function Calculation:

In general, the structural reliability analysis method is developed to solve a limit state function $g(X)$. Given the joint probability density function, $f_x(X)$, the probability of failure can be formulated as:

$$P_f = P(g \leq 0) = \int \ldots \int_\Omega f_x(\ X\ ) dx \qquad (4\text{-}7)$$

where Ω is the failure region. This multiple integral is in general very difficult to evaluate even though there is a Monte Carlo solution that can provide a convenient but usually time-consuming solution. The first step in the current reliability analysis methods requires the transformation of a general dependent, random vector **X** into an independent, standardized normal vector **u**. The Rosenblatt transformation [16] has been suggested for this, when the joint distribution is available [35,36]. If only the marginal distributions and the covariances are known, a transformation can be made to generate a joint normal distribution that satisfies the given correlation structure.

By transforming g(**X**) to g(**u**), the most probable point (MPP) in the u-space, **u***,is located. **u*** is the point that defines the minimum distance, β, from the origin (**u** = 0 point) to the limit state surface. This point is most probable because it has maximum joint probability density on the limit state surface, as shown in Figure 4-1. The MPP may be found by using optimization method or advanced mean value (AMV) method. Next, the g(**u**) or g(**X**) function is approximated by a polynomial that approximates the true function in the vicinity of the MPP. Once the approximate function is obtained, the associated failure probability can be computed. If the g(**u**) formulation is used, several analytical solutions
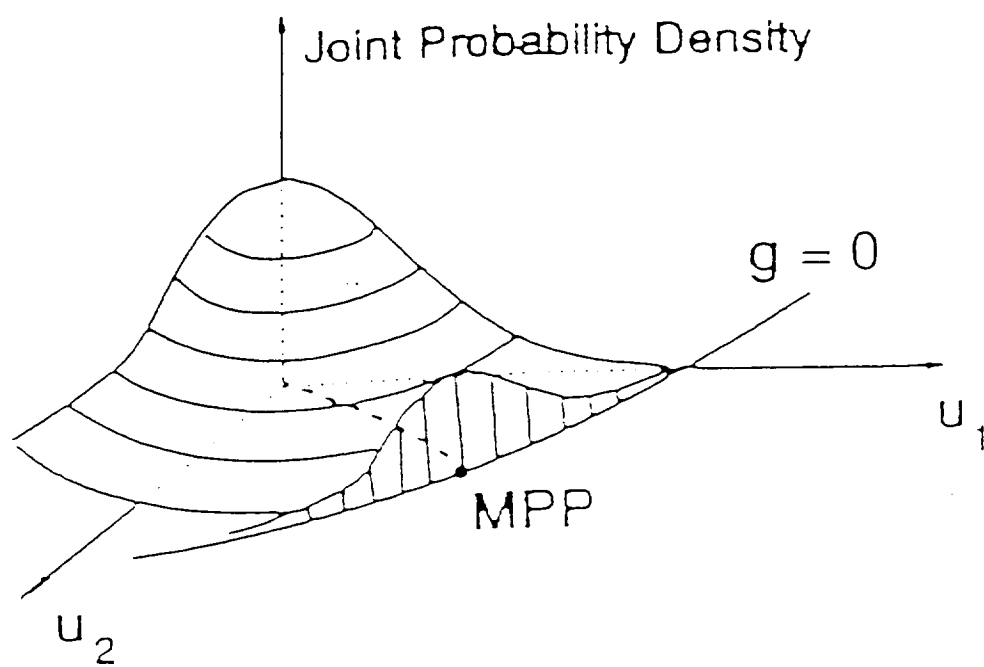
Figure 4-1 Illustration of a most probable
          point [28]

are available for linear and quadratic functions [36]. For example, the first-order reliability method (FORM) estimate is

$$P(g \leq 0) \approx \Phi (-\beta) \qquad (4-8)$$

To compute the component structural reliability for complex problems that require computation intensive programs, the Advanced Mean Value method (AMV) is suitable because it was developed to search for the MPP with fewest extra g function calculations by comparing with the conventional mean based second moment method [30,31,32].

Let us assume that the Taylor's series expansion of performance function, Z, exists at the mean values. The Z function can be expressed as:

$$
\begin{aligned}
Z(X) \quad &= Z(\mu) + \sum_{i-1}^{n} \frac{\partial Z}{\partial X_i} (X_i - \mu_i) + H(X) \\
&= \alpha_0 + \sum_{i-1}^{n} \alpha_i X_i + H(X) \\
&= Z_1(X) + H(X) \qquad (4-9)
\end{aligned}
$$

where the derivatives, α_i, are evaluated at the mean values, μ, Z_1 is a random variable representing the sum of the first-order terms, and H(X) represents the higher-order terms. In general,

the coefficients α_1 can be computed by numerical differentiation, and the minimum required number of Z-function calculations is (n+1) for n random variables. Since Z_1 is explicit and linear, its cdf (cumulative distribution function) can be computed efficiently.

For nonlinear Z-functions, the solution based on Z_1 is only approximate. To improve accuracy, higher-order approximation functions can be developed. However, for problems involving implicit Z-functions and a large n, the higher-order approach might be difficult and inefficient.

The AMV method reduces the truncation errors by replacing the higher-order terms H(**X**) by a simplified function $H(Z_1)$ dependent on Z_1. Ideally, the $H(Z_1)$ function should be based on the exact most probable point (MPP) locus of the Z function to minimize the truncation error. The AMV procedure simplifies this approach by using the MPP of Z_1.

At each calculated MPP, probability sensitivity factors, α, for every defined design random variables or random variables are the by-product from the reliability analysis. These sensitivity factors, as discussed, are defined in the transformed standard normal space (u - space):

$$\alpha = \frac{\partial \beta}{\partial u} = \frac{\partial \beta}{\partial \phi^{-1} (F_x(X))} \qquad (4\text{-}10)$$

where β represents the safety index value, Φ^{-1} represents the inverse standard normal cdf, and $F_x(X)$ represents the cdf for the original random variable, X. Comparing the absolute values for these sensitivity factors shows their relative importance to the reliability solution. If all design random variables have uncertain means (or standard deviations), the reliability itself becomes a random function of these uncertain parameters. To measure the effect caused by these uncertain parameters, the probabilistic sensitivity factors, with respect to these uncertain design parameters and reliability (safety index, β), can be derived as follows:

$$\frac{\partial \beta}{\partial \mu_x} = \frac{\partial \beta}{\partial u_x} \frac{\partial u_x}{\partial \mu_x} = \alpha \frac{\partial u_x}{\partial \mu_x} \qquad (4\text{-}11)$$

$$\frac{\partial \beta}{\partial \sigma_x} = \frac{\partial \beta}{\partial u_x} \frac{\partial u_x}{\partial \sigma_x} = \alpha \frac{\partial u_x}{\partial \sigma_x} \qquad (4\text{-}12)$$

where μ_x and σ_x represent mean and standard deviation values of random variable **X**, respectively. Since u_x is a function of μ_x and σ_x, $\partial u_x / \partial \mu_x$ and $\partial u_x / \partial \sigma_x$ can be derived also.

With these $\partial u_x / \partial \mu_x$ and $\partial u_x / \partial \sigma_x$ values evaluated, it is

possible to construct an approximate reliability constraint
function, $g_A(\mu_x, \sigma_x)$, as

$$g_A(\mu_x, \sigma_x) = \beta_0 + \sum_{j=1}^{n} \frac{\partial \beta}{\partial \mu_{x_j}} (\mu_{x_j} - \mu_{x_{j0}})$$

$$+ \sum_{k=1}^{n} \frac{\partial \beta}{\partial \sigma_{x_k}} (\sigma_{x_k} - \sigma_{x_{k0}}) - \phi^{-1}(1 - p)$$

$$= C_0 + \sum_{j=1}^{n} C_j \mu_{x_j} + \sum_{k=1}^{n} C_k \sigma_{x_k} - \phi^{-1}(1 - p) \qquad (4\text{-}13)$$

where β_0 is the safety index result, μ_{xj0} is the j-th initial
mean value, σ_{xk0} is the k-th initial standard deviation value,
$\Phi^{-1}(.)$ represents the inverse normal cumulative distribution
function (cdf),(1-p) is the select reliability goal, and C_0,
C_j, and C_k are constant.

Torng and Yang [28] have shown a safety index approximate
function $\beta_i(X)$ as :

$$\beta_i(X) = \beta_0 + \sum_{j=1}^{n} \frac{\partial \beta}{\partial \mu_{x_j}} (\mu_{x_j} - \mu_{x_{j0}})$$

$$+ \sum_{k=1}^{n} \frac{\partial \beta}{\partial \sigma_{x_k}} (\sigma_{x_k} - \sigma_{x_{k0}})$$

$$= C_0 + \sum_{j=1}^{n} C_j \mu_{x_j} + \sum_{k=1}^{n} C_k \sigma_{x_k} \qquad (4\text{-}14)$$

Therefore, an approximate reliability constrained function (or a limit state function), $g_A(X)$, can be defined as follows:

$$g_A(X) = C_0 + \sum_{j=1}^{n} C_j \mu_{x_j} + \sum_{k=1}^{n} C_k \sigma_{x_k} - \phi^{-1}(1 - p) \qquad (4\text{-}15)$$

or

$$g_A(X) = \beta_i(X) - \phi^{-1}(1 - p) \qquad (4\text{-}16)$$

In order to compute efficiently a reasonably accurate system sensitivity, the simplest and most efficient strategy is accomplished first by constructing an approximate function at the MPP of each bottom event. By using all approximate functions instead of the original complex failure models, the probability sensitivity, with respect to mean value and standard deviation, can be derived as $\partial u_x / \partial \mu_x$ and $\partial u_x / \partial \sigma_x$, respectively. This sensitivity can be calculated by perturbing all design random variables. Total computational effort is reduced greatly because these perturbation analyses are performed based on analytical approximate functions.

Wu, Torng, and Yang [28,31] point out AMV method is the best strategy for identifying the MPP for each of the failure models. Therefore, by using AMV, MPPs can be identified, and

approximate functions can be established. Once approximate functions are obtained, based on the objective function and all approximate functions, an optimization program is used to find the optimal design values. In this project GRG optimization program, which was developed by Dr. C. Onwubiko, is employed as an appropriate optimizer to obtain the optimal design values and safety index, the latter is based on the equation (2-2) with proper constrained functions.

4-5 Algorithm of Reliability Design Method Based on Optimization Design Method

Step 1. Define the optimal structure system or components requirement and construct an optimization design problem.

Step 2. Use the old design point as the initial design point. (In general, use the mean value of design random variables as the initial design point.)

Step 3. Construct an approximation function for component constraint function at the initial design point by the following steps:

  a). Evaluate the safety index , β_0, for the i-th reliability constraints at X_o by using advanced mean value (AMV) method or an appropriate optimizer.

  b). Construct an approximate constraint function for

component reliability constraint based on the safety

index with respect to the main values and standard

deviations of those design random variables.

c). Construct an approximate reliability constraint (or

limit state function).

d). Construct other approximation functions for other

constraint functions.

Step 4. Based on the objective function and all approximation

functions, GRG (GRG = Generalized Reduced Gradient

Method).optimizer is used to find the optimal design

values.

Step 5. Repeat steps 2 - 4 until the number of iteration is

reached or the convergence criterion is met.
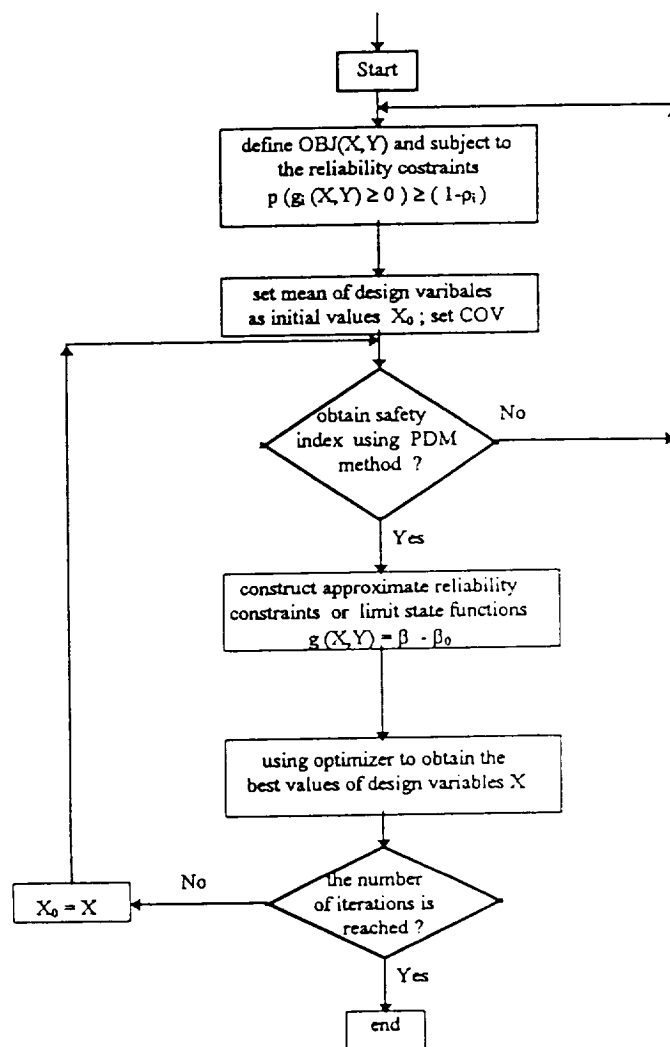
Figure. 5-2. Flow chart for reliability design
method based on optimization method

CHAPTER V

DESIGN OF A GEAR TRAIN USING A RELIABILITY

BASED OPTIMIZATION METHOD

5-1. Introduction

In engineering designs, the high reliability and minimum volume/cost/special design requirement of components / system is a goal pursued by engineers. A design of a gear or gear train is always considered an important and complex part of mechanical engineering design. Gears are often built into machines, e.g. as part of a gearbox. Smaller gears would imply a smaller gearbox, which leads to further savings. Tucker [37] says that maximizing load capacity for a given material and size generally results in lowest cost per horsepower transmitted. Willis [38] states that "weight reduction usually means volume reduction, which in turn lowers cost of materials, handling and shipping." It can be seen then that a good strategy is to minimize the size of the gear, not only because of direct saving on the gear, but also on related operations. Dudley [39] states, "It is often possible to reduce by half the length, width, and height of a gearbox by

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simply changing from steel gears with a low hardness value to full-hard gear teeth. This is an 8:1 reduction in gear weight, which means substantial savings in material, machinery, storage, and shipping costs of the gearing and the housing." In the automobile industry smaller gears mean lighter gears; hence, lighter gearboxes and ultimately lighter cars. The search for increased efficiency (i.e., fuel economy) makes reducing the size of gears important regardless of initial cost. In the case of helicopters, reduction in size and weight can result in an increase in payload [40].

This chapter describes the minimization of the rotation output of the gear system, a special design requirement, using reliability based optimization method.

5-2. Model Formulation

In designing gears, there are at least two major causes of failure models that are of primary concern: bending and contact stress. A gear train must satisfy the rotation output objective as well as the system reliability constraints, which are used to account for uncertainty existing in different failure models, material properties, and geometric parameters of gears. According to optimization design methods, the

deterministic problem is stated as

Minimize $\quad F_v(\mathbf{X}) = n_{in} \prod (X_i / X_{i+1})$

$$i = 1, 2, 3, \ldots, M \qquad\qquad (5\text{-}1)$$

subject to

$$\frac{W_t P_i}{K_v B_i J_i} - \sigma_{b\,i} \leq 0 \qquad\qquad (5\text{-}2)$$

$$\sqrt{\frac{W_t}{\cos\theta\,\pi\,B_i}\,\frac{\dfrac{1}{R_{P\,i}} + \dfrac{1}{R_{G\,i}}}{\dfrac{(1 - \mu_{P\,i}^2)}{E_{P\,i}} + \dfrac{(1 - \mu_{G\,i}^2)}{E_{G\,i}}}} - \sigma_{c\,i} \leq 0 \qquad\qquad (5\text{-}3)$$

$$X_l \leq \mathbf{X} \leq X_u ; \quad N_l \leq \mathbf{X} \leq N_u ;$$

where

M = number of gears and pinions in system.

B_i = gear face width (in). ($B_i = \lambda X_i$)

P_i = diametral pitch (number of tooth/in).

X_i = pitch diameter for gears and pinions (in).

λ = coefficient of gear face width.

θ = the pressure angle.

W_t = the transmitted load.

J_i = geometry factor of gear and pinion.

K_v = dynamic factor.

μ_{Pi} = poisson's ratio of pinion.

μ_{Gi} = poisson's ratio of pinion.

E_{Pi} = modules of elasticity for pinion.

E_{Gi} = modules of elasticity for pinion.

σ_{bi} = allowable bending stresses.

σ_{ci} = allowable bending stresses.

R_{pi} = radius of involute on pinion.

R_{Gi} = radius of involute on gear.

n_{in} = input of rotation speed (1/min)

N_i = number of tooth for gears and pinions $(N_i = D_i \, p_i)$

X, N are a column vector with n rows and the subscripts l and u represent the lower and upper bounds on X, N respectively.

Because the probabilistic design is concerned with probability of failure or the reliability of system, the probabilistic equivalent formulation of (5-1);(5-2);(5-3) can be written as:

$$\text{Minimize} \quad F_v(\mathbf{X}) = n_{in} \prod (X_i \, / \, X_{i+1})$$

$$i = 1, 2, 3, \ldots, M \qquad\qquad (5\text{-}4)$$

subject to

$$P[G_i(X) \le 0] \ge \rho_i$$

$$i = 1, 2, 3, \ldots, M \qquad (5\text{-}5)$$

where

$$G_i(X) = \frac{W_i \, P_i}{K_v \, B_i \, J_i} - \sigma_{b\,i} \qquad (5\text{-}6)$$

$$G_{i+1}(X) = \sqrt{\frac{W_i}{\cos\theta \, \pi \, B_i} \, \frac{\dfrac{1}{R_{P\,i}} + \dfrac{1}{R_{G\,i}}}{\dfrac{(1 - \mu_{P\,i}^2)}{E_{P\,i}} + \dfrac{(1 - \mu_{G\,i}^2)}{E_{G\,i}}}} - \sigma_{c\,i} \qquad (5\text{-}7)$$

and **X** is a vector of n random variables and ρ_i is the specified reliability level of the system.

In terms of the principle of reliability design method based on optimization techniques, the formulation given in equations (5-4),(5-5),(5-6) and (5-7) were recast for application of reliability based optimization. They were

$$\text{Minimize} \quad F_v(X) = n_{in} \prod (X_i \, / \, X_{i+1})$$

$$i = 1, 2, 3, \ldots, M \qquad (5\text{-}8)$$

subject to

$$G_i(\mathbf{X}) = \beta_i - \phi^{-1}(\rho_i)$$

$$i = 1,2,3,\ldots,M \qquad\qquad (5\text{-}9)$$

where $G_i(\mathbf{X})$ are defined by equations (5-6) and (5-7) and $\phi^{-1}(.)$ is the inverse of the standard normal distribution function. Of course, it may be necessary to scale (5-9) to avoid problem when using nonlinear program for constrained optimization of the type presented in [28][32].

The mean value of the pitch diameter of teeth in the pinions and gears is to be determined for a minimum rotation output of a gear train to satisfy some specified reliability level. It is assumed that all material properties reported are at their mean values. Since actual data are generally not available, the standard deviation, σ, may be estimated by coefficient of variation.

$$COV = \frac{\sigma_i}{\overline{X}_i} \qquad\qquad (5\text{-}10)$$

where COV is the coefficient of variation, σ_i is the standard deviation, and X is the mean value of a random variable.

5-3 Example Design

To demonstrate the application of reliability based optimization method, we consider the following problem:

Designing a spur gear train shown on Figure 5-1 involves minimizing the rotation output while satisfying the stress constraints. It is delivered transmitting 100 hp with a shaft rotating input at 2000 rmp. The material to be used is AISI 1095. The material properties and other information are given in Table 5-1.

To execute this design problem, certain assumptions are made. The dynamic factor k_v is assumed to be 1. The J-factor is computed using the fitted equation given by Carrol and Johnson [41]. Because of the limitations of design geometric and undercutting, N_1 is assumed to be 17, and N_u is assumed to be 100. Since the maximum contact stress occurs at the lowest point of single tooth contact [42], this point is close to the pitch point; thus, the sliding velocity is small.

Therefore, the formula (5-7) is modified as follows:

$$\sqrt{\frac{2\,W_t}{B_t}\left(\frac{X_{P_t} + X_{G_t}}{X_{P_t}X_{G_t}}\right)\frac{1}{\frac{1-\mu^2}{E_{P_t}} + \frac{1-\mu^2}{E_{G_t}}}} - \sigma_d \leq 0 \qquad (5\text{-}11)$$

2

3

2000 rpm
input

1

4
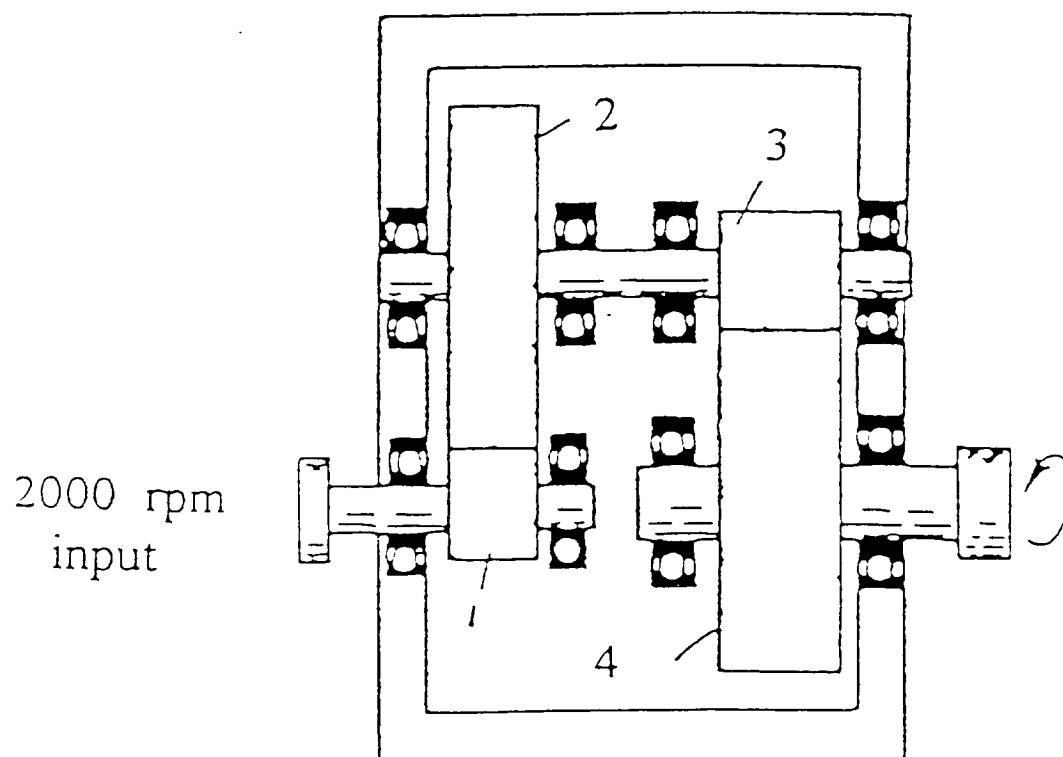
Figure 5-1. Gear train for design example [43]

Table 5 - 1. Summary of material properties and other information for the example problem

Variables	Values
Power (pw)	100 hp
Rotation input	2000 (1/min)
Yield stress σ_{y1}	83 kpsi (572.4 MPa)
Yield stress σ_{y2}	83 kpsi (572.4 MPa)
Tensile stresss σ_{T1}	142 kpsi (979.3 MPa)
Tensile stresss σ_{T2}	142 kpsi (979.3 MPa)
Possion's ratio μ	0.30
Pressure angle θ	20^0
Coefficient of width λ	0.60
Coefficient of variation (COV)	0.05
Power efficiency η	99.0 %
Modulus of elasticity E_{p1} ; E_{p2}	30×10^6 Psi (205 Gpa)
Modulus of elasticity E_{g1} ; E_{g2}	30×10^6 Psi (205 Gpa)

where in terms of Figure 5-2

$$\frac{1}{R_{P_i}} + \frac{1}{R_{G_i}} = \frac{1}{r_{P_i}\sin\phi} + \frac{1}{r_{G_i}\sin\phi} \qquad (5\text{-}12)$$

and

$$r_{P_i} = \frac{X_{P_i}}{2}; \qquad r_{G_i} = \frac{X_{G_i}}{2} \qquad (5\text{-}13)$$

therefore

$$\frac{1}{R_{P_i}} + \frac{1}{R_{G_i}} = \frac{2}{\sin\phi} \left(\frac{X_{P_i} + X_{G_i}}{X_{P_i} X_{G_i}}\right) \qquad (5\text{-}14)$$

After the modification of functions, then GRG (Generalized Reduced Gradient method) optimizer is applied to calculating the probability of failure, reliability and safety index. The probability of failure, reliability, and safety index are illustrated in Table 5-2, which are based on the calculation using GRG computer program.
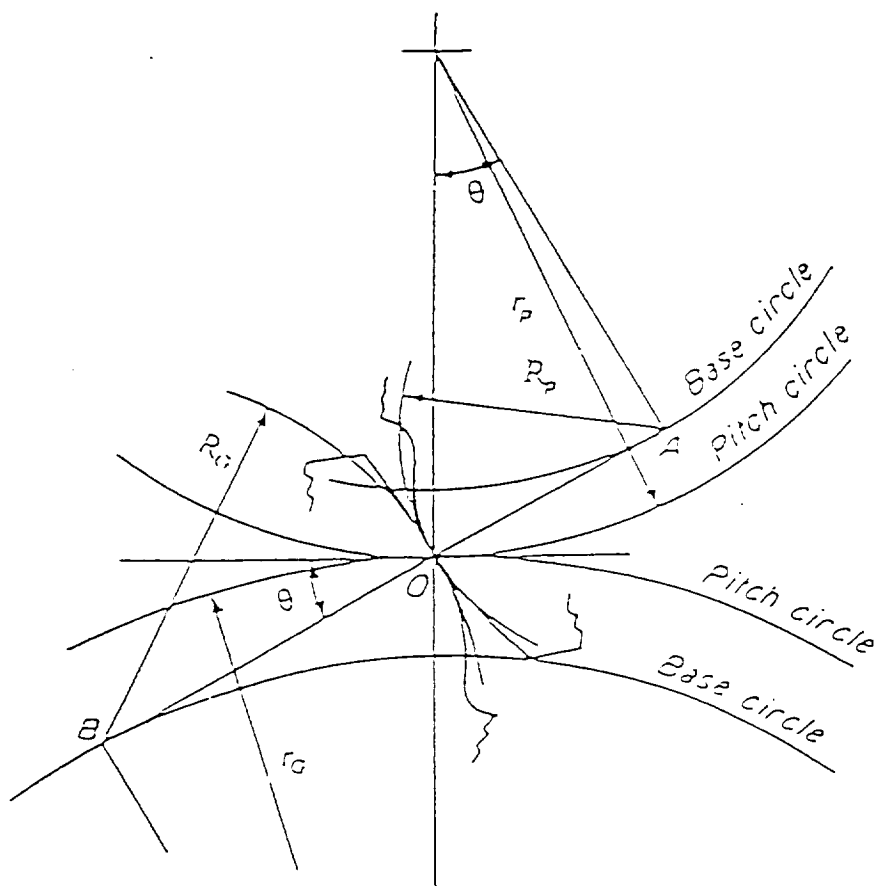
Figure 5-2. Radii of curvature R_P and R_G for tooth surface at pitch point O

This system can be considered to be series system. A series system is one in which all components are so interrelated that the entire system will fail even if any one of its components fails. Let us suppose that the components are independent, namely, that the performance of any one part does not affect the reliability of the others. Under these conditions, the reliability of this system is defined as follows:

$$R_s = \prod_{i=1}^{n} R_i \qquad (5\text{-}15)$$

where R_s is the reliability of system, R_i is the reliability of each component, and i is equal to 4 for this design.

The approximate function can be constructed for the computation of optimization method. It is

$$G_i(\mathbf{X}) = \beta_i - \phi^{-1}(\rho_i)$$
$$i = 1,2 \qquad (5\text{-}16)$$

where $G_i(\mathbf{X})$ is defined by (5-6), (5-11) and ρ_i is defined by the reliability of system R_s.

After the construction of the approximate functions, an appropriate optimizer, GRG computer program is used in order to obtain the best design results for this design system.

Optimization functions are based on objective function (5-8) and constraints (5-16). These values, rotation input n_{in}; Possion's ratio μ_i; Modulus of elasticity E_i; coefficient of width λ_i; delivered transmitting hp, are kept constant during the optimization process. Diametral pitch p_i are assumed as 6 (teeth/in) and 5 (teeth/in). The results for this design system using reliability based optimization methods are shown in Tables 5-2 through 5-5 and Figures 5-3 through 5-7. All of calculation for this project in detail is in Appendix A.

Table 5 - 2.   Results for the reliability of each component

Situation	Prob.of failure P_f	Reliability R_i (%)	safety index β_i
bending case for pinion 1 $g_1(X)$ function(Eq.5-2,i=1)	0.15900×10^{-3}	99.9841	3.599571
contact case for pinion 1 $g_2(X)$ function(Eq.5-3,i=1	0.70500×10^{-5}	99.999295	4.3433106
bending case for pinion 2 $g_3(X)$ function)Eq.5-2,i=2)	0.54810×10^{-2}	99.4519	2.543948
contact case for pinion 2 $g_4(X)$ function(Eq.5-3,i=2)	0.72686×10^{-2}	99.273136	2.443667
system	0.12874×10^{-1}	98.7126265	2.230000

Table 5 - 3. Results for reliability vs. the change of pitch diameter for each gear, rotation output (System safety index $\beta = 2.23$.)

reliablity (%)	d_1, pitch diameter(in)	d_2, pitch diameter(in)	d_3, pitch diameter(in)	d_4, pitch diameter(in)	rotation output(1/min)
*	4.03	15.28	6.35	14.98	223.71
93.32	-	-	-	-	-
96.41	3.12	14.11	5.24	13.87	166.93
97.13	3.28	14.46	5.48	14.17	175.73
97.73	3.49	14.80	5.75	14.47	187.43
97.98	3.60	14.95	5.90	14.62	194.33
98.715	4.03	15.28	6.35	14.98	223.71
98.78	4.08	15.39	6.41	15.02	226.15
98.81	4.11	15.42	6.45	15.04	228.60
99.18	-	-	-	-	-

Note: * indicates deterministic solution .
       - shows no feasible solution .

Table 5 - 4. Results for reliability vs. the change of the number of teeth, face width
(System safety index β = 2.23.)
(Assumed the diametral pitch P₁ = 6 teeth/in; P₂ =5 teeth/in)

reliablity (%)	N₁ , number of teeth	N₂ , number of teeth	N₃ , number of teeth	N₄ , number of teeth	face width for pinion 1 (in)	face width for pinion 2 (in)
*	24	92	32	75	2.4	3.81
93.32	-	-	-	-	-	-
96.41	19	85	26	70	1.87	3.14
97.13	20	87	27	71	1.97	3.29
97.73	21	89	29	72	2.09	3.45
97.98	22	90	30	73	2.16	3.54
98.715	24	92	32	75	2.40	3.81
98.78	25	92	32	75	2.45	3.85
98.81	25	93	32	75	2.47	3.87
99.18	-	-	-	-	-	-

Note: * indicates deterministic solution .
       - shows no feasible solution .

Table 5 - 5. Results for reliability vs. applied stresses and rotation output
(System safety index β = 2.23.)

reliablity (%)	bending stress for pinion 1 (ksi)	contact stress for pinion 1 (ksi)	bending stress for pinion 2 (ksi)	contact stress for pinion 2 (ksi)	rotation output (1/min)
*	10.07	79.46	12.17	80.20	223.71
93.32	-	-	-	-	-
96.41	13.10	78.65	16.06	79.00	166.93
97.13	12.23	78.90	14.96	79.37	175.73
97.73	11.38	79.14	13.83	79.72	187.43
97.98	10.99	79.24	13.27	79.88	194.33
98.715	10.07	79.46	12.17	80.20	223.71
98.78	9.98	79.48	12.16	80.22	226.15
98.81	9.90	79.50	12.05	80.23	228.60
99.18	-	-	-	-	-

Note: * indicates deterministic solution .
       - shows no feasible solution .
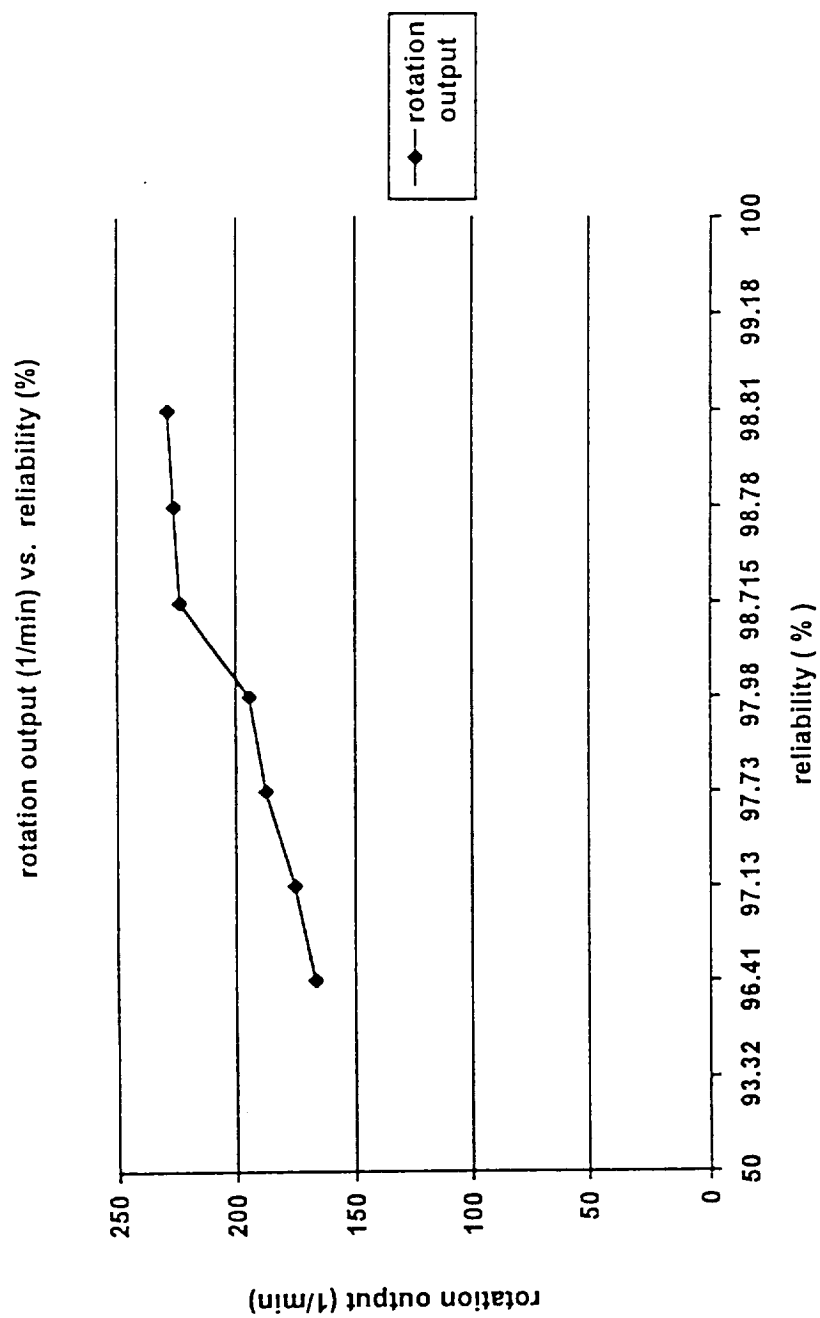
Figure 5-3. The comparison of rotation output vs. relaibility
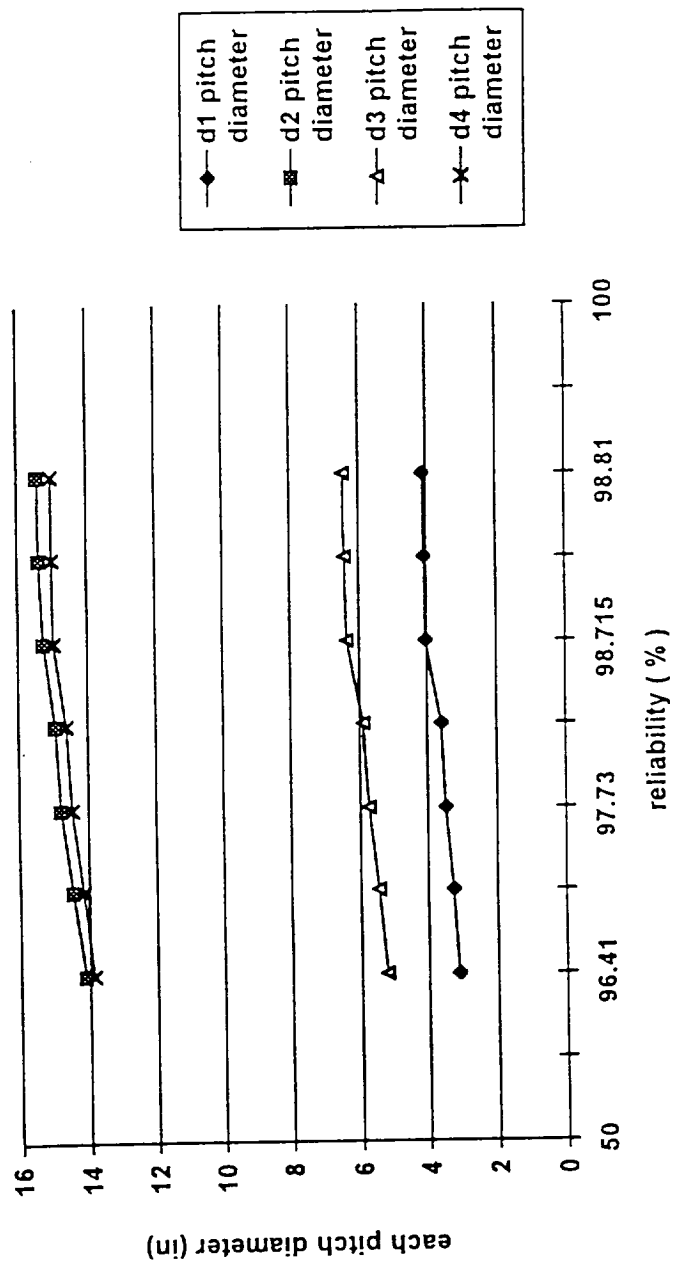
each pitch diameter (in) vs. reliability

Figure 5-4. The comparison of each pitch diameter vs. reliability

the changes of the number of tooth vs. reliability
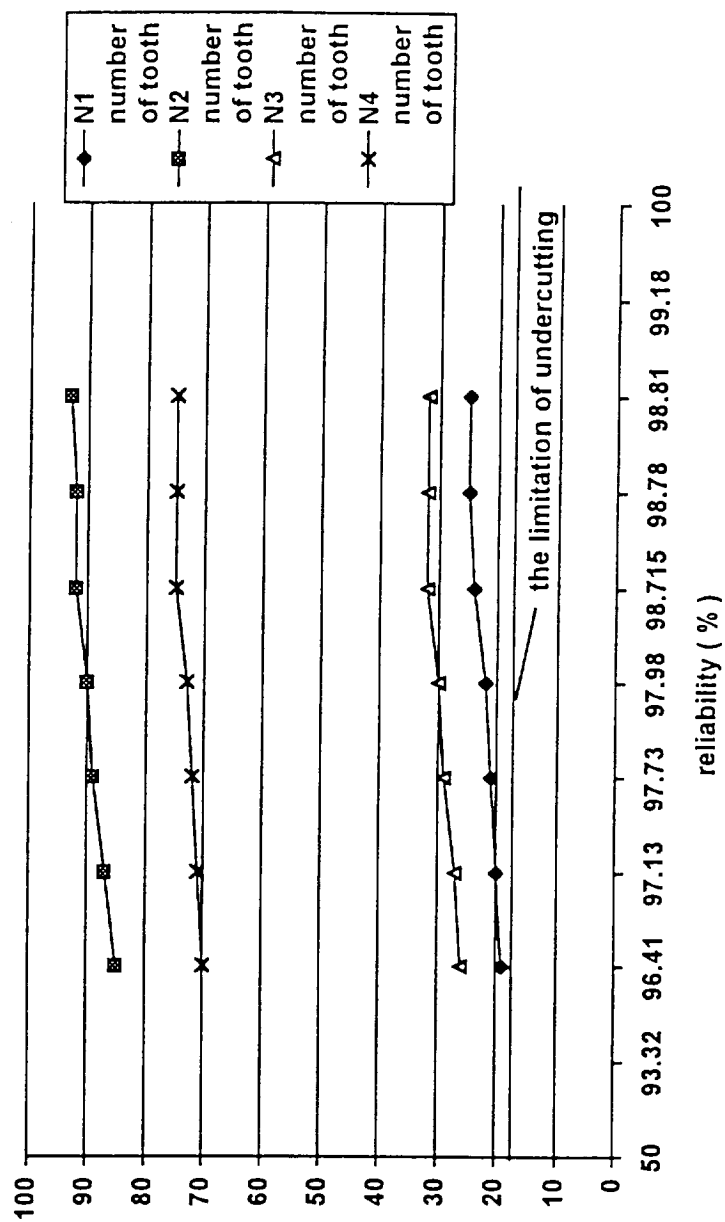

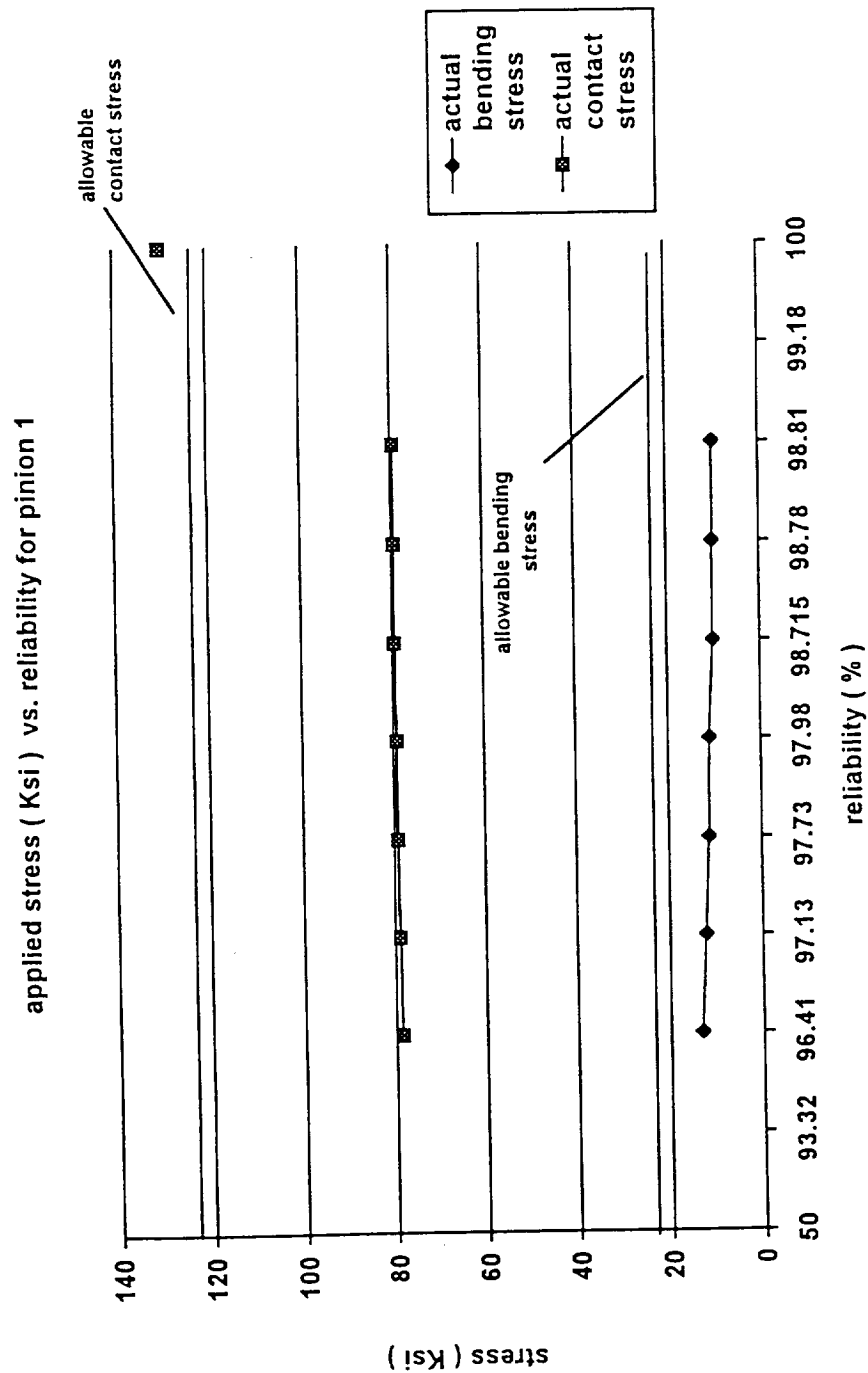Figure 5-5. The comparison of the changes of number of tooth vs.reliability

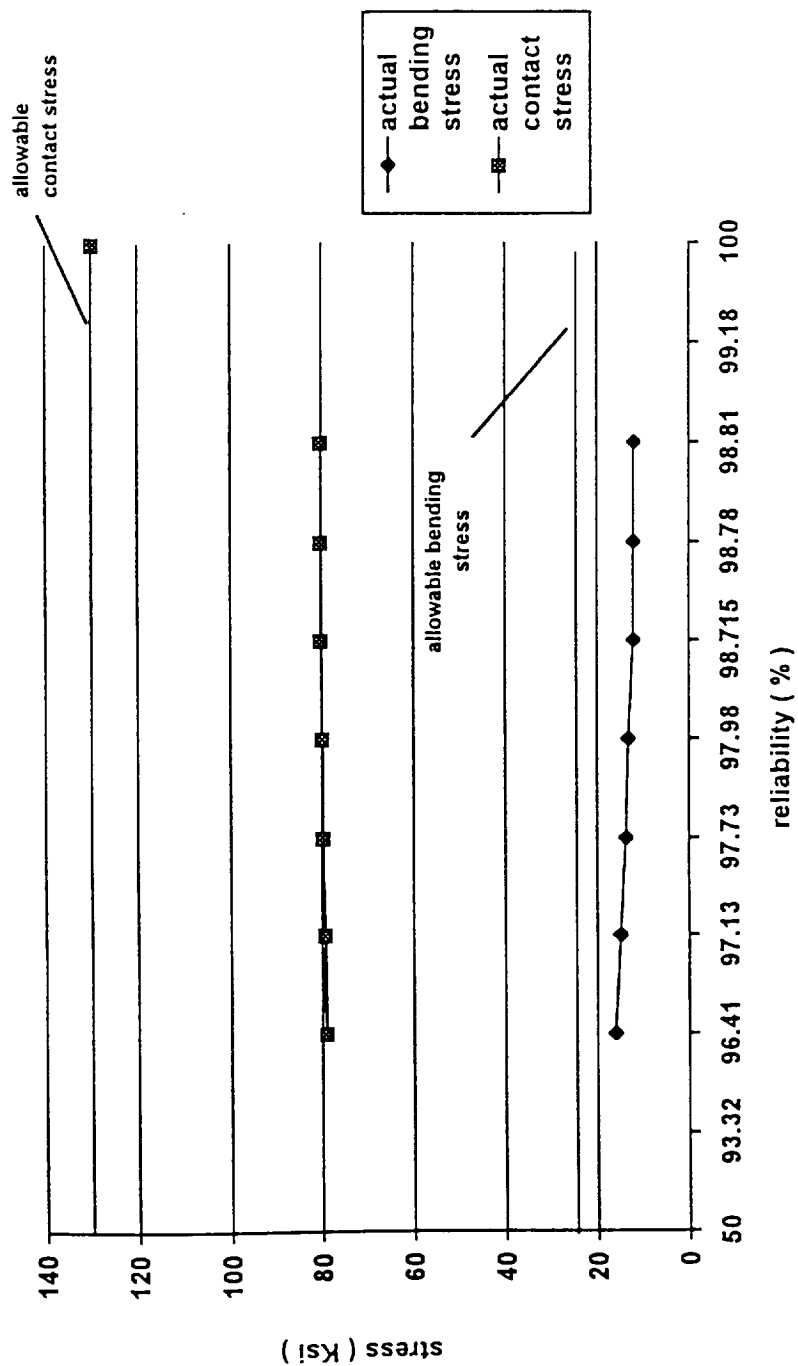Figure 5-6. The comparison of applied stress (Ksi) vs. reliability

Figure 5-7. The comparison of applied stress (ksi) vs. reliability (pinion 2)

5-4. Discussions

To aid in the discussion of the research, the actual stresses in the pinion tooth are shown in Figures 5-6 and 5-7. The system safety index shown in Tables 5-3 through 5-5 is the minimum possible, based on the given information in Table 5-1. By comparing the results, the rotation output of the gear train is increasing when the system reliability is increasing. This means if the higher reliability of system in design is selected, the heavier, larger system has to be taken. However, there is the limitation of a system reliability taken by a designer. It is impossible to obtain higher reliability of system more than 99.379 percent in this design. The reason is that reliability for an engineering system depends on the mean and standard deviation or Coefficient of Variation (COV) of design parameters. According to the principle of Probabilistic Design Method (PDM), if the higher standard deviation or COV in design is used, then the system will have the suitable reliability. Once this reliability exceeded the limitation of design situation, all of the design parameters are infeasible as shown in Figures 5-3 through 5-7.

From the Tables 5-3 through 5-5, the values of the design variables, which obtained by using Optimization Method and Reliability Design Method based on optimization techniques, are the same in the system reliability of 98.715 percent. It

is true that the deterministic values are the same as the probabilistic approach. By transforming the limit state functions g(**X**) to g(u) that had been mentioned previously, the most probable point (MPP) in the u-space is on the objective and constraint functions. Therefore, when the same safety index is taken, the values of design variables obtained by using both methods should be the same.

Finally, we can see from the Figures 5-6 and 5-7, with the increase of reliability, the bending stress tends to decreasing while the contact stress inclines to increasing. This situation indicates that the failure of contact stress is more sensitive than the failure of bending stress. In conclusion, the design mainly has to be concentrated on the contact stress when trying to deliver high power and high rotation in the design of gear.

CHAPTER VI

CONCLUSIONS AND FUTURE RESEARCH SUGGESTIONS

6-1. Conclusions

The Probabilistic Design Method (PDM) is widely used in engineering design. PDM can be employed for stochastic design parameters to obtain the values of design variables under a specific reliability of component / system. PDM eliminates the deterministic design method's defect that the design variables must be deterministic. Also, this method makes a wider range of the values of design variables that can be selected by design engineers. However, if the design objective function is minimized for the volume or cost of component / system or a special design requirement, the values of design variables obtained by using PDM could not be the optimal design points in the design. Furthermore, this method reaches the failure of probability or a reliability of component / system, which is based on the mean and standard deviation of random design parameters.

Optimization design method is a powerful tool in

engineering design. It emphasizes how to obtain the values of design variables that make the design objective function minimum or maximum using mathematical tools. The values of design variables obtained using optimization design method are optimal and critical; in other words, these values of design variables must make the system or components higher reliability or lower probability of failure. However, optimization design methods belong to a deterministic design method. In practice, design variables cannot be considered deterministic but stochastic. In addition, in the deterministic approach, random effects are ignored.

There is no question that both PDM and optimization design method have their own disadvantages. Probabilistic design method was not concerned with the minimum or maximum design objectives but the probability of failure or the reliability of a system. In optimization design method, design variables must be deterministic. The method of reliability based on optimization design method eliminates these disadvantages with PDM and optimization design method. Not only that it can be applied to the situation of uncertain design variables in engineering design but also provide the wide range of reliability that designers can choose for engineering system or components in optimization design.

6-2 Suggestions for Future Research:

Based on the results of the present study, the following topics may be addressed in future research:

In this design of gear train, only both bending and contact stress were taken as design limit functions. Actually, it is complex to practically design a gear train, especially when the input power is more than 75 KW. Therefore, various design factors may be considered in future design of a gear train, such as thermal conditions, wear and dynamic factors, and so on.

Since the safety index is defined as the minimum distance from the origin to the surface of the limit state function, minimizing the safety index using optimization techniques is a kind of calculated method. Therefore, there may be the comparison of using optimization techniques and NESSUS code to compute safety index and reliability of each component and system in future research.

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APPENDICES

APPENDIX A:

A-1. The functions for calculating the safety index β

Objective function:

$$safety\ index\ (\beta) = \left(\sum_{i=1}^{n} x_i^2\right)^{\frac{1}{2}}$$

where n = number of design variables in function (slack variables are not included)

Constrained functions:

1) Bending function (Equation 5-2, i = 1)

G(1)= 3.49*(1+ 0.05* x(6))-
      (1+0.05*x(5))*(1.76*x(1)*x(2)*x(3)+17.36*x(2)+6.68*x(1))
      $\div$ (x(1)^3 x(2)*x(4))-x(7)^2
G(2)= 1.75-x(5)-x(8)^2
G(3)= x(6)+1.8-x(9)^2
G(4)= x(7)
G(5)= x(8)
G(6)= x(9)

where x(1)= pitch diameter of pinion 1.   x(2)= pitch diameter of gear 1.
      x(3)= gear face width.   x(4)= diametral pitch.
      x(5)= the transmitted load.   x(6)= allowable bending stress.
      x(7),x(8),x(9) are slack variables.

2) Bending function (Equation 5-2, i = 2)

G(1)= 4.005*(1+ 0.05* x(6))-
      x(8)*(1+0.05*x(5))*(1.76*x(1)*x(2)*x(3)+17.36*x(2)+6.68*x(1))
      $\div$ (x(1)^3 x(2)*x(4)*x(7))-x(9)^2
G(2)= 1.75-x(5)-x(10)^2
G(3)= x(6)+1.8-x(11)^2
G(4)= x(9)
G(5)= x(10)
G(6)= x(11)

where x(1)= pitch diameter of pinion 2.   x(2)= pitch diameter of gear 2.
    x(3)= gear face width.   x(4)= diametral pitch.
    x(5)= the transmitted load.   x(6)= allowable bending stress.
    x(7)= pitch diameter of pinion 1.   x(8)= pitch diameter of gear 1.
    x(9),x(10),x(11) are slack variables.

3)   Contact function (Equation 5-3, i = 1)

$G(1) = 0.169*(1 + 0.05*x(5))^2 -$
$(1 + 0.05*x(4))*2.079*(x(1) + x(2)) \div (x(1)^3 \ x(3)*x(2)) - x(6)^2$
$G(2) = 1.75 - x(4) - x(7)^2$
$G(3) = x(5) + 1.32 - x(8)^2$
$G(4) = x(6)$
$G(5) = x(7)$
$G(6) = x(8)$

where x(1)= pitch diameter of pinion 1.   x(2)= pitch diameter of gear 1.
    x(3)= gear face width.   x(4)= the transmitted load.
    x(5)= allowable contact stress.   x(6),x(7),x(8) are slack variables.

4)   Contact function (Equation 5-3, i = 2)

$G(1) = 0.169*(1 + 0.05*x(5))^2 -$
$(1 + 0.05*x(4))*2.058*x(6)*(x(1) + x(2)) \div (x(1)^3 \ x(3)*x(2)*x(7)) - x(8)^2$
$G(2) = 1.75 - x(4) - x(9)^2$
$G(3) = x(5) + 1.32 - x(10)^2$
$G(4) = x(8)$
$G(5) = x(9)$
$G(6) = x(10)$

where x(1)= pitch diameter of pinion 2.   x(2)= pitch diameter of gear 2.
    x(3)= gear face width.   x(4)= the transmitted load.
    x(5)= allowable contact stress.   x(6)= pitch diameter of pinion 1.
    x(7)= pitch diameter of gear 1.   x(8),x(9),x(10) are slack variables.

A-2. Input data for calculation of the safety index β

items	Bending Function 1	Bending Function 2	Contact Function 1	Contact Function 2
x(1)	-2	-1	-1	-1
x(2)	-1	-1	-1	-1
x(3)	1	-1	-1	-1
x(4)	-1	1	1	2
x(5)	1	1	-1.5	-1.5
x(6)	-1.5	-1.5	1	-1
x(7)	1	-1	1	1
x(8)	1	1	1	1
x(9)	1	1		1
x(10)		1		1
x(11)		1		
slack variables	x(7),x(8),x(9)	x(9),x(10),x(11)	x(6),x(7),x(8)	x(8),x(9),x(10)

A-3. output data from calculation of the safety index

items	Bending Function 1	Bending Function 2	Contact Function 1	Contact Function 2
x(1)	-2.04827	-0.2909197	-2.871211	-0.7595228
x(2)	-0.8867294	-0.1438399	-1.516372	0.7614115
x(3)	0.1829933	-1.20627E-2	-1.874681	8.755293E-2
x(4)	-1.280323	0.2049768	1.75000	1.747578
x(5)	1.75000	1.749835	-1.32000	-1.318175
x(6)	-1.8000	-1.799837	-5.4607E-11	-0.0891021
x(7)	-6.9628E-11	-0.1511611	-1.8506E-11	8.755323E-2
x(8)	-1.6310E-10	8.280489E-5	3.6605E-11	-7.770096E-7
x(9)	-3.0381E-11	3.369595E-7		-1.522156E-5
x(10)		-1.19630E-7		-1.863136E-6
x(11)		-2.82766E-7		
safety index β	3.599571	2.543948	4.343106	2.443667

A-4 The functions for calculating the optimal values of design variables

Objective function:

$$Rotation\ output\ F(X) = 2000 \times \frac{x(1)\ x(3)}{x(2)\ x(4)}$$

Constrained functions:

G(1)=1-10.51÷(x(5)*x(1)^3 *x(2))
    *(10.58*x(1)*x(2)+17.36*x(2)+6.68*x(1)) +2.23-β
G(2)=1-10.4*x(2)÷(x(6)*x(1)*x(3)^3 *x(4))
    *(8.815*x(3)*x(4)+17.36*x(4)+6.68*x(3) +2.23-β
G(3)=1-588.467÷x(7)*((x(1)+x(2))÷(x(1)^3 *x(2)))$^{0.5}$ +2.23-β
G(4)=1-585.517÷x(8)*(x(2)*(x(3)+x(4))÷(x(3)^3 *x(1)*x(4)))$^{0.5}$ +2.23-β

where   x(1)= pitch diameter of pinion 1.   x(2)= pitch diameter of gear 1.
        x(3)= pitch diameter of pinion 2.   x(4)= pitch diameter of gear 2.
        x(5)= applied bending stress for bending function 1.
        x(6)= applied bending stress for bending function 2 .
        x(7)= applied contact stress for contact function 1.
        x(8)= applied contact stress for contact function 2.
        β   = specific safety index selected by designer.

A-5   input data for calculation of optimal design variables
        (assumed the diametral pitch of the first pair of gears = 6 1/in; the diametral pitch
        of the second pair of gears = 5 1/in)

items	x(1)	x(2)	x(3)	x(4)	x(5)	x(6)	x(7)	x(8)
initial points	3.7	9.7	5	11	22	25	120	125
limited maximum	16.7	16.7	20	20	83	83	142	142
limited minimum	2.8	2.8	3.4	3.4	5	5	5	5

note: unit for x(1) to x(4): in.
        unit for x(5) to x(8): ksi.

A-6 output data from calculation of optimal design variables (system safety index β=2.23)

specific safety index β	x(1)	x(2)	x(3)	x(4)	x(5)	x(6)	x(7)	x(8)	F(x)
β = 1.5	-	-	-	-	-	-	-	-	-
β = 1.8	3.12	14.1	5.24	13.9	13.1	16.1	78.7	79.0	166.9
β = 1.9	3.28	14.5	5.48	14.2	12.2	15.0	78.9	79.4	175.7
β = 2.0	3.49	14.8	5.75	14.5	11.4	13.8	79.1	79.7	187.4
β = 2.05	3.60	15.0	5.90	14.6	11.0	13.3	79.2	79.9	194.3
β = 2.23	4.03	15.3	6.34	15.0	10.1	12.4	79.4	80.1	223.6
β = 2.25	4.08	15.4	6.41	15.0	10.0	12.2	79.5	80.2	226.2
β = 2.26	4.11	15.4	6.45	15.0	9.90	12.1	79.5	80.2	228.6
β = 2.28	-	-	-	-	-	-	-	-	-

note: - means infeasible solution
      unit for F(X): 1/min.
      unit for x(1) to x(4): in.
      unit for x(5) to x(8): ksi.

APPENDIX B: LIST OF GRG COMPUTER PROGRAM

```
DECLARE SUB FTERMIN (maintest!, x!(), n!, ne!, pvalue!)
DECLARE SUB CONSTRAINT (x!(), cons!(), n!, ne!)
DECLARE SUB PENALFX (x!(), cons!(), fxvalue!, pvalue!, beta!, n!)
DECLARE SUB QNEWTON (x!(), s!(), n!, tol!)
DECLARE SUB LINESEARCH (x!(), s!(), n!, h!, tol!)
DECLARE SUB UPDATEDFP ()

DECLARE SUB TERMINATION ()
DECLARE FUNCTION OBJECTIVE! (x!(), n!)
DECLARE SUB DIRECTION ()
DECLARE SUB UPDATEMAT ()
DECLARE SUB PDERIVATIVE ()
DIM SHARED x(20), xo(20), y(20), f(20), p(20), u(20), ds(20), dr(20, 20)
DIM SHARED delx(20), delg(20), s(20), grad(20)
COMMON SHARED ja, jb, jc, jd, jmin, xmin, ji, fbase, iter
COMMON SHARED ql, tol, fmin, G, l, n, m, size, c1, fcount, lgec
COMMON SHARED mloop, fec, d, fo, h, gtest, start, hstep, update
COMMON SHARED pvalue, ne, beta
CLS
PRINT
PRINT
INPUT "How many variables"; n
INPUT "Total number of constraints"; ne
DIM SHARED b(n, n), cons(ne)
INPUT "what is the tolerance"; tol
d = .1: fcount = 0: fec = 0: mloop = 1
 DIM SHARED xt(n), ao(n, n), gx(n)
FOR i = 1 TO n
   PRINT "x("; i; ")": INPUT x(i)
   xo(i) = x(i): y(i) = x(i)
NEXT i
INPUT "enter output file please:", outfile$
IF outfile$ = "" THEN END
OPEN outfile$ FOR OUTPUT AS #2
beta = 1: maintest = 1
DO UNTIL maintest = 0
   CALL QNEWTON(x(), s(), n, tol)
   CALL FTERMIN(maintest, x(), n, ne, pvalue)
   beta = 1.5 * beta
   FOR i = 1 TO n: PRINT "x("; i; ")="; x(i): NEXT i
```

```
        PRINT " pvalue="; pvalue, beta: ' INPUT o
LOOP
        finish = TIMER
          PRINT "finish="; finish
        maxtime = finish - sstart

        finish! = TIMER
        PRINT #2, "Program took"; maxtime; " sec."; giter; "iteration(s)"
        PRINT
PRINT #2, "The solution after"; iter; "iteration(s) :"
FOR i = 1 TO n
    PRINT #2, "x("; i; ") ="; x(i); "grad("; i; ")="; grad(i)
NEXT i
  CALL PENALFX(x(), cons(), fxvalue, pvalue, beta, n)
  fo = OBJECTIVE(x(), n)
  PRINT #2, "Objective Function="; fo, "fcount="; fcount
  IF update = 1 THEN
    PRINT #2, "Using the DFP update and initial step of"; cstep
  ELSE
    PRINT #2, "Using the BFGS update and initial step of"; cstep
  END IF
PRINT #2, "The solution after"; iter; "iteration(s) :"
FOR i = 1 TO n
    PRINT #2, "x("; i; ") ="; x(i); "grad("; i; ")="; grad(i)
NEXT i
  PRINT #2, "Objective Function="; fo, "fcount="; fcount
  IF update = 1 THEN
    PRINT #2, "Using the DFP update and initial step of"; cstep
  ELSE
    PRINT #2, "Using the BFGS update and initial step of"; cstep
  END IF
PRINT #2, "tolerance="; tol
END

SUB BFGS
DIM at(n), af(n, n)
  FOR i = 1 TO n
    delg(i) = grad(i) - gx(i)
    delx(i) = x(i) - xo(i)
  NEXT i
'Compute first denuminator
denum1 = 0
FOR i = 1 TO n
```

```
    denum1 = denum1 + delx(i) * delg(i)
NEXT i
' Compute the second denominator
denum2 = 0
FOR j = 1 TO n
    sum = 0
    FOR k = 1 TO n
        sum = sum + delg(k) * ao(k, j)
    NEXT k
    denum2 = denum2 + sum * delg(j)
NEXT j
IF denum1 <> 0 AND denum2 <> 0 THEN
    FOR i = 1 TO n
        FOR j = 1 TO n
            b(i, j) = delx(i) * delx(j) / denum1
        NEXT j
    NEXT i
        FOR j = 1 TO n
            sum1 = 0
            FOR k = 1 TO n
            sum1 = sum1 + ao(j, k) * delg(k)
            NEXT k
            at(j) = sum1
        NEXT j
        FOR i = 1 TO n
            FOR j = 1 TO n
            af(i, j) = at(i) * at(j) / denum2
            NEXT j
        NEXT i
' Form updated matrix
    FOR i = 1 TO n
        FOR j = 1 TO n
        ao(i, j) = ao(i, j) + b(i, j) - af(i, j)
        NEXT j
    NEXT i: start = 1
ELSE
    start = 0
END IF
END SUB

SUB CONSTRAINT (x(), cons(), n, ne)
REM contact function 1
'cons(1) = .169 * (1 + .05 * x(5)) ^ 2 - (1 + .05 * x(4)) * 2.079 * ((x(1) + x(2)) / (x(1) ^ 3 * x(3)
```

```
* x(2))) - x(6) ^ 2
'cons(2) = 1.75 - x(4) - x(7) ^ 2
'cons(3) = x(5) + 1.32 - x(8) ^ 2
'cons(4) = x(6)
'cons(5) = x(7)
'cons(6) = x(8)

REM bending function 2
cons(1) = 4.005 * (1 + .05 * x(6)) - x(8) * (1 + .05 * x(5)) * (1.76 * x(1) * x(2) * x(3) + 17.36 *
x(2) + 6.68 * x(1)) / (x(1) ^ 3 * x(2) * x(4) * x(7)) - x(9) ^ 2
cons(2) = 1.75 - x(5) - x(10) ^ 2
cons(3) = x(6) + 1.8 - x(11) ^ 2
cons(4) = x(9)
cons(5) = x(10)
cons(6) = x(11)

REM bending function 1
'cons(1) = 3.49 * (1 + .05 * x(6)) - (1 + .05 * x(5)) * (1.76 * x(1) * x(2) * x(3) + 17.36 * x(2) +
6.68 * x(1)) / (x(1) ^ 3 * x(2) * x(4)) - x(7) ^ 2
'cons(2) = 1.75 - x(5) - x(8) ^ 2
'cons(3) = x(6) + 1.8 - x(9) ^ 2
'cons(4) = x(7)
'cons(5) = x(8)
'cons(6) = x(9)

REM contact function 2
'cons(1) = .169 * (1 + .05 * x(5)) ^ 2 - (1 + .05 * x(4)) * 2.06 * x(6) * ((x(1) + x(2)) / (x(1) ^ 3 *
x(3) * x(2) * x(7))) - x(8) ^ 2
'cons(2) = 1.75 - x(4) - x(9) ^ 2
'cons(3) = x(5) + 1.32 - x(10) ^ 2
'cons(4) = x(8)
'cons(5) = x(9)
'cons(6) = x(10)
END SUB

SUB DIRECTION
IF start = 0 THEN
'Set the identity matrix
    FOR i = 1 TO n
        FOR j = 1 TO n
            IF i = j THEN
                ao(i, j) = 1
            ELSE
```

```
            ao(i, j) = 0
         END IF
       NEXT j
     NEXT i
END IF
'Identify point at which A is set to identiy matrix
    lgec = gec
' Save the initial point and the gradient at this point
FOR j = 1 TO n
    xo(j) = x(j)
    gx(j) = grad(j)
NEXT j
' Form the step and product of step and gradient
    c1 = 0: snorm = 0
  FOR i = 1 TO n
     sum = 0
     FOR j = 1 TO n
        sum = sum - ao(i, j) * grad(j)
     NEXT j
        s(i) = sum: c1 = c1 - sum * grad(i)
        snorm = snorm + s(i) ^ 2
  NEXT i
  'Check if normalization is necessary
  IF snorm > 100 THEN
       snorm = snorm ^ .5
       FOR i = 1 TO n
          s(i) = s(i) / snorm
       NEXT i
  END IF
END SUB

SUB FTERMIN (maintest, x(), n, ne, pvalue)
 IF pvalue <= tol OR beta > 1E+20 THEN
    maintest = 0
 ELSE
    CALL PDERIVATIVE
    gradvalue = 0
    FOR i = 1 TO n
       gradvalue = gradvalue + grad(i) ^ 2
    NEXT i
    gradvalue = gradvalue ^ .5
    IF gradvalue <= tol THEN
       maintest = 0
```

```
     END IF
   END IF
 END SUB

SUB LINESEARCH (x(), s(), n, h, tol)
  DIM x1(n)
  tol1 = .0001
  CALL PENALFX(x(), cons(), fxvalue, pvalue, beta, n)
  f1 = fxvalue
  FOR t = 1 TO n
      x1(t) = x(t)
  NEXT t
  FOR t = 1 TO n
   x(t) = x1(t) + h * s(t)
  NEXT t
  CALL PENALFX(x(), cons(), fxvalue, pvalue, beta, n)
  ff = fxvalue
  IF ff < f1 THEN
      route1 = 1
      DO UNTIL route1 = 0
         f2 = ff:
         FOR t = 1 TO n
             x(t) = x1(t) + 2 * h * s(t):
         NEXT t
  CALL PENALFX(x(), cons(), fxvalue, pvalue, beta, n)
         ff = fxvalue
         IF ff > f2 THEN
           route1 = 0: f3 = ff
         ELSE
           h = 2 * h
         END IF
      LOOP
  ELSE
      route1 = 1
      DO UNTIL route1 = 0
         f3 = ff:
         FOR t = 1 TO n
           x(t) = x1(t) + .5 * h * s(t):
         NEXT t
       CALL PENALFX(x(), cons(), fxvalue, pvalue, beta, n)
           ff = fxvalue
         IF ff < f1 THEN
           route1 = 0: f2 = ff: h = h / 2
```

```
            ELSE
                h = -.5 * h:  ' Note, you may change sign to "+"
            END IF
            IF ABS(h) <= 1E-14 THEN
                route1 = 0
            END IF
        LOOP
    END IF
IF h > 1E-14 THEN
    d = .5 * h * (4 * f2 - 3 * f1 - f3) / (2 * f2 - f1 - f3)
    a = 0: b = h: c = 2 * h
    test1 = 1
    DO UNTIL test1 = 0
        FOR t = 1 TO n
            x(t) = x1(t) + d * s(t):
        NEXT t
            CALL PENALFX(x(), cons(), fxvalue, pvalue, beta, n)
                f4 = fxvalue
        'Check convergence
            IF ABS(f2 - f4) <= toll OR ABS(b - d) <= toll THEN
                IF f4 < f2 THEN
                    alpha = d
                ELSE
                    alpha = b
                END IF
                FOR t = 1 TO n
                    x(t) = x1(t) + alpha * s(t)
                    PRINT "x("; t; ")="; x(t)
                NEXT t
            CALL PENALFX(x(), cons(), fxvalue, pvalue, beta, n)
                fopt = fxvalue
                PRINT "fopt="; fopt
                test1 = 0
            ELSE
                ' Check that bracket is not lost in discarding the max. pt
                IF a <= d AND d <= b THEN
                    IF f1 >= f4 AND f4 <= f2 THEN
                        c = b: f3 = f2: b = d: f2 = f4
                    ELSE
                        a = d: f1 = f4
                    END IF
                ELSE
                    IF f2 >= f4 AND f4 <= f3 THEN
```

```
                    a = b: f1 = f2: b = d: f2 = f4
                ELSE
                    c = d: f3 = f4
                END IF
            END IF
        END IF
    num = (b ^ 2 - c ^ 2) * f1 + (c ^ 2 - a ^ 2) * f2 + (a ^ 2 - b ^ 2) * f3
    den = (b - c) * f1 + (c - a) * f2 + (a - b) * f3
        IF den = 0 THEN
            d = 0
        ELSE
            d = .5 * num / den
        END IF
    LOOP
ELSE
    FOR t = 1 TO n
        x(t) = x1(t)
    NEXT t
END IF
END SUB


FUNCTION OBJECTIVE (x(), n)
OBJECTIVE = (x(1) ^ 2 + x(2) ^ 2 + x(3) ^ 2 + x(4) ^ 2 + x(5) ^ 2 + x(6) ^ 2 + x(7) ^ 2 + x(8) ^
2) ^ .5
END FUNCTION


SUB PDERIVATIVE
DIM z1(100)
' Subroutine computes the gradient
    delta = .001: status = 1
    FOR j = 1 TO n
        z1(j) = x(j)
        x(j) = z1(j) + delta
            CALL PENALFX(x(), cons(), fxvalue, pvalue, beta, n)
                y1 = fxvalue
        x(j) = z1(j) - delta
            CALL PENALFX(x(), cons(), fxvalue, pvalue, beta, n)
                y2 = fxvalue
        grad(j) = (y1 - y2) / (2 * delta)
        x(j) = z1(j)
    NEXT j
END SUB
```

```
SUB PENALFX (x(), cons(), fxvalue, pvalue, beta, n)
CALL CONSTRAINT(x(), cons(), ne, n)
    sumc = 0
    FOR i = 1 TO ne
      sumc = sumc + cons(i) ^ 2
    NEXT i
    pvalue = sumc ^ .5:
' Handle the bounds on variables
    fxvalue = OBJECTIVE(x(), n)
    fxvalue = fxvalue + beta * (sumc)
END SUB


SUB QNEWTON (x(), s(), n, tol)
   status = 1: pass = 0: iter = 0
   hstep = 1: mloop = 1: cstep = hstep
'Evaluate gradient at current point x()
   CALL PDERIVATIVE:   gec = gec + 1
'Start iteration process
    start = 0
    sstart = TIMER
        CALL PENALFX(x(), cons(), fxvalue, pvalue, beta, n)
           fo = fxvalue
             fec = fec + 1
DO UNTIL mloop = 0
   CALL DIRECTION
' Test for search direction
        CALL PENALFX(x(), cons(), fxvalue, pvalue, beta, n)
          fbase = fxvalue
   ' Perform a line search
   '----------------------------
      h = hstep
    CALL LINESEARCH(x(), s(), n, h, tol)
       CALL PENALFX(x(), cons(), fxvalue, pvalue, beta, n)
          fmin = fxvalue
     iter = iter + 1
          CALL PDERIVATIVE
          CALL TERMINATION:  'Check for convergence
      IF mloop <> 0 THEN
         IF fmin >= fbase THEN
           start = 0:  ' Restart the search using old direction
     '                but reduce step size
             hstep = h / 2
             IF h <= .000001 * cstep THEN
```

```
                mloop = 0
            END IF
        ELSE
            CALL UPDATEDFP
     'Set a new search direction
            hstep = cstep
        END IF
      END IF
 LOOP
END SUB

SUB TERMINATION
  xtest = 0: ftest = 0: gtest = 0: gtol = .000001
  ftol = .000001: xtol = .000001
  FOR i = 1 TO n
     'IF ABS(xo(i) - x(i)) <= xtol THEN
        xtest = xtest + ABS(xo(i) - x(i))
   ' END IF
     gradvalue = gradvalue + (grad(i)) ^ 2
  NEXT i: gtest = (gradvalue) ^ .5
  IF fbase <> 0 THEN
     ftest = (fmin - fbase)
  END IF
    IF gtest <= gtol THEN
       mloop = 0
    END IF
END SUB

SUB UPDATEDFP
DIM at(n), af(n, n)
  FOR i = 1 TO n
     delg(i) = grad(i) - gx(i)
     delx(i) = x(i) - xo(i)
  NEXT i
'Compute first denuminator
denum1 = 0
FOR i = 1 TO n
  denum1 = denum1 + delx(i) * delg(i)
NEXT i
' Compute the second denominator
denum2 = 0
FOR j = 1 TO n
   sum = 0
```

```
      FOR k = 1 TO n
         sum = sum + delg(k) * ao(k, j)
      NEXT k
      denum2 = denum2 + sum * delg(j)
   NEXT j
   IF denum1 <> 0 AND denum2 <> 0 THEN
      FOR i = 1 TO n
         FOR j = 1 TO n
            b(i, j) = delx(i) * delx(j) / denum1
         NEXT j
      NEXT i
         FOR j = 1 TO n
            sum1 = 0
            FOR k = 1 TO n
            sum1 = sum1 + ao(j, k) * delg(k)
            NEXT k
            at(j) = sum1
         NEXT j
         FOR i = 1 TO n
            FOR j = 1 TO n
             af(i, j) = at(i) * at(j) / denum2
            NEXT j
         NEXT i
   ' Form updated matrix

      FOR i = 1 TO n
         FOR j = 1 TO n
         ao(i, j) = ao(i, j) + b(i, j) - af(i, j)
         NEXT j
      NEXT i: start = 1
   ELSE
      start = 0
   END IF
   END SUB

   SUB UPDATEMAT
   DIM AA(n, n), af(n, n)
    FOR i = 1 TO n
         delg(i) = grad(i) - gx(i)
         delx(i) = x(i) - xo(i)
    NEXT i
   denum = 0
    FOR i = 1 TO n
```

```
        denum = denum + delx(i) * delg(i)
    NEXT i
IF denum <> 0 THEN
 FOR i = 1 TO n
    FOR j = 1 TO n
      IF i = j THEN
        b(i, j) = 1 - (delx(i) * delg(j)) / denum
      ELSE
        b(i, j) = -(delx(i) * delg(j)) / denum
      END IF
      NEXT j
  NEXT i
  FOR i = 1 TO n
    FOR j = 1 TO n
      sum = 0
      FOR k = 1 TO n
        sum = sum + b(i, k) * ao(k, j)
      NEXT k
      af(i, j) = sum
    NEXT j
  NEXT i
  FOR i = 1 TO n
    FOR j = 1 TO n
      sum = 0
      FOR k = 1 TO n
        sum = sum + af(i, k) * b(k, j)
      NEXT k
        AA(i, j) = sum + (delx(i) * delx(j)) / denum
        ao(i, j) = AA(i, j)
    NEXT j
  NEXT i
  FOR i = 1 TO n
    FOR j = 1 TO n
    NEXT j
  NEXT i: start = 1
ELSE
    start = 0
END IF
END SUB
```